Lecture # 21

Warm-up:

1) Compute the value of $\sin(\pi/4 + \alpha)$ where $\sin(\alpha) = 1/3$ and $\tan(\alpha) < 0$.

$\hookrightarrow \tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$

$\hookrightarrow \quad \sin(s+t) = \sin(s)\cos(t) + \sin(t)\cos(s).$

$$\Rightarrow \sin(\pi/4 + \alpha) = \sin(\pi/4)\cos(\alpha) + \sin(\alpha)\cos(\pi/4).$$

$$= \frac{\sqrt{2}}{2} \cdot \cos(\alpha) + \frac{\sqrt{2}}{2}\sin(\alpha)$$

$$= \frac{\sqrt{2}}{2} \cdot \cos(\alpha) + \sqrt{2}/6.$$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\Rightarrow \cos(\alpha) = -\sqrt{8/9} = -\sqrt{8}/3.$$

$$\Rightarrow \sin(\pi/4 + \alpha) = \frac{-\sqrt{2} \cdot \sqrt{8}}{6} + \frac{\sqrt{2}}{6}.$$

2) If $\sin(t) = -2/3$ and t lies in the third quadrant, then what is $\sin(2t)$?

$\hookrightarrow \quad \sin(2x) = 2\sin(x)\cos(x)$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\Rightarrow \sin(2t) = 2\sin(t)\cos(t).$$

$$= \frac{-4}{3} \cdot \cos(t).$$

$$(-2/3)^2 + \cos^2(t) = 1$$

$$\cos(t) = -\sqrt{5/9} = -\sqrt{5}/3.$$

$$\sin(2t) = 4\sqrt{5}/9.$$

Fact: (Formulas for lowering powers)

1) $$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

2) $$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

↳ Follow from the double angle formulas.

Ex: Simplify the expression so there are no exponents.

$\sin^2(x)\cdot\cos^2(x)$

$$= \frac{1-\cos(2x)}{2}\cdot\frac{1+\cos(2x)}{2}$$

$$= \frac{1-\cos^2(2x)}{4}$$

$$= \frac{1-\frac{1+\cos(4x)}{2}}{4}$$

$$= \frac{1}{4} - \frac{1+\cos(4x)}{8}$$

$$= \frac{1}{4} - \frac{1}{8} - \frac{\cos(4x)}{8}$$

$$= \frac{1}{8} - \frac{\cos(4x)}{8}$$

Ex: Simplify the expression $\tan^2(2x)$ so there are no exponents.

$$\tan^2(2x) = \frac{\sin^2(2x)}{\cos^2(2x)} = \frac{1-\cos(4x)}{1+\cos(4x)}.$$

1) $\sin^2(y) = \dfrac{1-\cos(2y)}{2}$ $\overset{y=2x}{\Rightarrow}$ $\sin^2(2x) = \dfrac{1-\cos(4x)}{2}$

2) $\cos^2(y) = \dfrac{1+\cos(2y)}{2} \Rightarrow \cos^2(2x) = \dfrac{1+\cos(4x)}{2}$

Rmk: Other formulas => See HW / Book.

Fact:

$$\sin^2(x/2) = \frac{1-\cos(x)}{2} \Rightarrow \sin(x/2) = \pm\sqrt{\frac{1-\cos(x)}{2}}$$

$$\cos^2(x/2) = \frac{1+\cos(x)}{2} \Rightarrow \cos(x/2) = \pm\sqrt{\frac{1+\cos(x)}{2}}$$

Ex: Compute $\cos(\pi/8) = \cos((\pi/4)/2)$

$$= \pm\sqrt{\frac{1+\cos(\pi/4)}{2}}$$

$$= \pm\sqrt{\frac{1+\sqrt{2}/2}{2}}$$

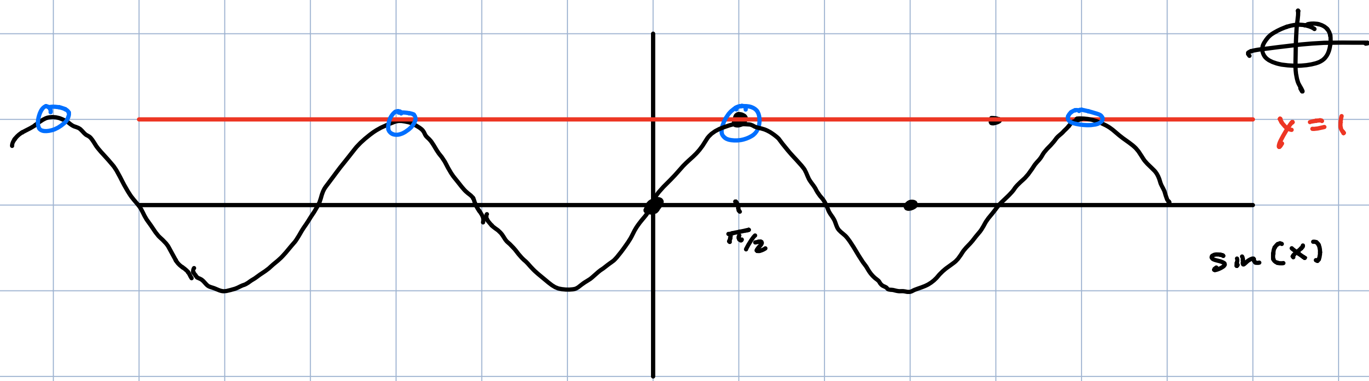
Note $\pi/8$ lies in I quad

$\Leftrightarrow \cos(\pi/8) \geq 0$

$\Rightarrow \cos(\pi/8) = +\sqrt{\dfrac{1}{2} + \dfrac{\sqrt{2}}{4}}$

Section 7.4 – 7.5: Trig Eqns.

Ex: Solve for all real values of x that satisfy $\sin(x) = 1$.



$\Rightarrow$ there are infinitely many solutions.

Note, $x = \pi/2$ is a soln.

$\Rightarrow$ $x = \pi/2 + 2\pi$ is a soln

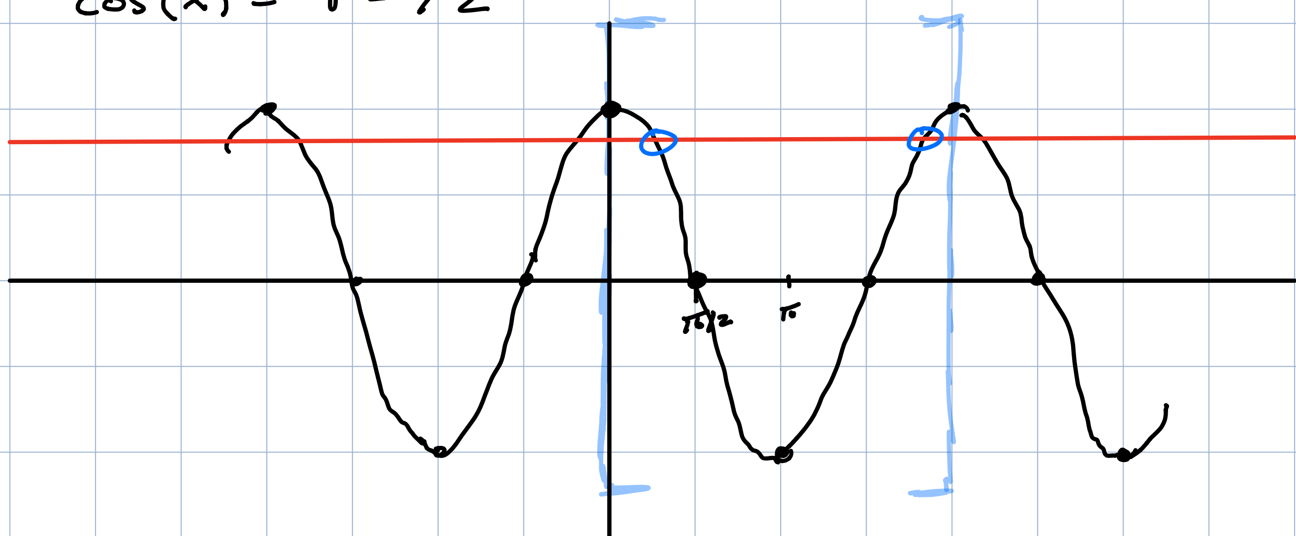
$x = \pi/2 + 4\pi$ " "

$x = \pi/2 - 2\pi$ " "

$n = -3, -2, -1, 0, 1, 2, 3, \ldots$

So $x = \pi/2 + 2\pi \cdot n$ where n is an integer

Ex: $\cos(x) = \sqrt{2}/2$



What values of x between $0 \le x < 2\pi$ satisfy $\cos(x) = \sqrt{2}/2$?

Note, $\cos(\pi/4) = \sqrt{2}/2 \Rightarrow x = \pi/4$ is a soln!

$\cos(7\pi/4) = \cos(-\pi/4) = \sqrt{2}/2$

$\Rightarrow x = 7\pi/4$ is a soln!

$\Rightarrow x = \pi/4 + 2\pi n$ for n an integer.

or

$x = 7\pi/4 + 2\pi n$ 〃 〃 〃 〃

Strategy: For $\text{trig}(x) = c$ (think $\sin(x) = c$ or $\cot(x) = c$).

1) Find all solns over a repeated interval of the graph of the trig fcn

2) Deduce (via periodicity) that all soln will be the particular solns from ① plus integer multiples of the period.

Trig Fcn	① Particular Interval	② Period Multiples
sin	$[0, 2\pi)$	$2\pi \cdot n$
cos	$[0, 2\pi)$	$2\pi \cdot n$
sec	〃	〃
csc	〃	〃
tan	$(-\pi/2, \pi/2)$	$\pi \cdot n$
cot	$(0, \pi)$	$\pi \cdot n$

Ex: Find all real soln to $\tan(x) = -\sqrt{3}$.

① Start by finding all $-\pi/2 < x < \pi/2$ st $\tan(x) = -\sqrt{3}$.

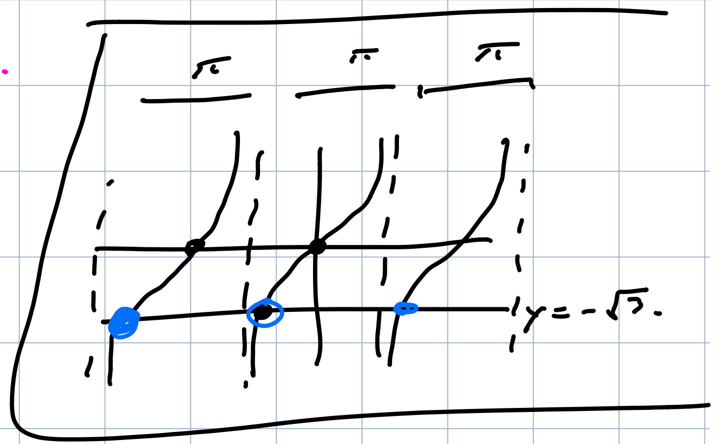
$x = 2\pi/3$ is a sol

$x = -\pi/3$ is a soln.

Graphically, we see this is the only soln in $(-\pi/2, \pi/2)$.

② $\Rightarrow$ all soln are $x = -\pi/3 + \pi n$

$-\pi/3 + \pi = 2\pi/3$.



Ex: Trig eqns of quadratic type

$$\sin^2(x) + 2\sin(x) - 3 = 0 \xleftrightarrow{y = \sin(x)} y^2 + 2y - 3 = 0$$

$$(\sin(x) + 3)(\sin(x) - 1) = 0 \qquad (y+3)(y-1) = 0$$

$\Rightarrow$ ① $\sin(x) = -3$ <u>or</u> ② $\sin(x) = 1$.

① $\sin(x) = -3$.

$\hookrightarrow$ range of sin is $[-1, 1]$

$\Rightarrow$ $\sin(x) \neq -3$ for all values of x.

$e^x = -1$

$\hookrightarrow$ range of e^x is $(0, +\infty)$.

② $\sin(x) = 1 \Rightarrow x = \pi/2 + 2\pi n$.

$\Rightarrow$ The original eqn has soln $x = \pi/2 + 2\pi n$.