Lecture #2

Recall: Last time ⇝ review arithmetic; add, sub, fraction, mult, etc.

This time ⇝ powers of #s and roots

⇝ Algebraic expression

Defn: $a^n = \underbrace{a \cdot \ldots \cdot a}_{n\text{-times}}$, where a = real #, n is nat. # >0

↪ $2^3 = 2 \cdot 2 \cdot 2 = 4 \cdot 2 = 8$

↪ $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

↪ $3^3 = 27$

a is called the base and n is called the exponent

Defn: i) $a^0 = 1$ when $a \neq 0$

ii) $a^{-n} = \frac{1}{a^n}$

↪ $42^0 = 1$

↪ $7777^0 = 1$

↪ $2^{-2} = \frac{1}{2^2} = \frac{1}{2 \cdot 2} = \frac{1}{4}$

↪ $\left(\frac{2}{3}\right)^{-3} = \frac{1}{(2/3)^3} = \frac{1}{\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}} = \frac{1}{8/27} = \frac{27}{8}$

Rule: i) $a^n \cdot a^m = a^{n+m}$

↪ $2^2 \cdot 2^3 = 2^5 = 32$

$(2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 4 \cdot 8 = 32$ ✓

ii) $a^n / a^m = a^n \cdot a^{-m} = a^{n-m}$

↪ $2^2 / 2^3 = 2^{2-3} = 2^{-1} = \frac{1}{2}$ ✓

$4/8 = 1/2$

iii) $(a^m)^n = a^{m \cdot n}$

$\hookrightarrow (2^2)^2 = (2 \cdot 2)^2 = (2 \cdot 2)(2 \cdot 2) = 2^4$

iv) $(ab)^n = a^n \cdot b^n$

$\hookrightarrow (2 \cdot 7)^2 = 2^2 \cdot 7^2 = 4 \cdot 49 = 196$

$14^2 = 196$

Rmk: Similar rules for fractions.

Ex: Simplify the following:

$$\left(\frac{x}{y}\right)^3 \cdot \left(\frac{y^2 x^2}{xz}\right)^{-2} = \frac{x^3}{y^3} \cdot \left(\frac{y^2 x^2}{xz}\right)^{-2}$$

$$= \frac{x^3}{y^3} \cdot \left(\frac{x^1 z}{y^2 x^2}\right)^2$$

$$= \frac{x^3}{y^3} \cdot \frac{x^2 z^2}{(y^2)^2 (x^2)^2}$$

$$= \frac{x^3}{y^3} \cdot \frac{x^2 z^2}{y^4 \cdot x^4}$$

$$= \frac{x^5 z^2}{y^7 \cdot x^4}$$

$$= xz^2/y^7.$$

Ex: Simp. $\frac{6st^{-3}}{(s^{-2}t^4)^3} = \frac{6st^{-3}}{(s^{-2})^3 \cdot (t^4)^3}$

$$= \frac{6st^{-3}}{s^{-6}\, t^{12}}$$

$$= \frac{6s}{s^{-6} t^{12} \cdot t^{3}}$$

$$= \frac{6s}{s^{-6} \cdot t^{15}}$$

$$= \frac{6s \cdot s^{6}}{t^{15}}$$

$$= 6s^{7}/t^{15}$$

Defn: The principal n^{th} root, $\sqrt[n]{a} = b$ st $b^{n} = a$

↪ this called radical notation

↪ If $n =$ even, then $a \geq 0$, $b \geq 0$.

↪ $\sqrt{-1} = b \Rightarrow b^{2} = -1 \rightsquigarrow b$ does not exist.

↪ $\sqrt{4}$; $2^{2} = 4$, $(-2)^{2} = 4$

Ex: $\sqrt[4]{81} = 3$; $3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 = 81$ ✓

$\sqrt{4^{2}} = 4$; $4 \cdot 4 = 4^{2} = 16$

$\quad \shortparallel$

$\sqrt{16}$

Rule: i) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

↪ $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$ ✓

$\quad \shortparallel$

$\sqrt{36} = 6$

ii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n \cdot m]{a}$

↪ $\sqrt[2]{\sqrt[2]{81}} = \sqrt[4]{81} = 3$

$\sqrt[n]{9} = 3$

iv) when n is odd $\sqrt[n]{a^n} = a$

v) " " " even, $\sqrt[n]{a^n} = |a|$

↳ $n = 2$, $a = -2$;

$\sqrt{(-2)^2} = \sqrt{4} = 2 = |-2|$

$n = 3$, $a = -2$;

$\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$

$(-2 \cdot (-2) \cdot (-2) = -8)$

Ex: $\sqrt[7]{\dfrac{a^3 b^2}{a^{10} \cdot \sqrt{b}}} = \sqrt[7]{\dfrac{b^2}{a^7 \sqrt{b}}}$

$\dfrac{a^3}{a^{10}} = a^{3-10} = a^{-7} = \dfrac{1}{a^7}$

$= \dfrac{\sqrt[7]{b^2}}{\sqrt[7]{a^7}\,\sqrt[7]{\sqrt{b}}}$

$= \dfrac{\sqrt[7]{b^2}}{a\,\sqrt[7]{\sqrt{b}}}$

$= \sqrt[7]{b^2} / ((a) \cdot (\sqrt[14]{b}))$

$\frac{2}{4}$ is not lowest terms; $\frac{2}{4} = \frac{1}{2}$

Defn: m/n in lowest terms, $n > 0$,

$a^{m/n} = (\sqrt[n]{a})^m$

↳ $m = 1$; $a^{1/n} = \sqrt[n]{a}$

↳ All the previous rules for exponentials hold for these fractional exponents

↳ $a^{m/n} \cdot a^{k/l} = a^{\frac{m}{n} + \frac{k}{l}}$.

↳ $n =$ even, $a > 0$ ⇝ $\sqrt[n]{a}$ $n =$ even $\Rightarrow a > 0$.

Ex: i) $125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \dfrac{1}{5}$

ii) $(a^3 \cdot b^4)^{3/2} = a^{9/2} \cdot b^6 = \sqrt{a^9} \cdot b^6$

Ex: Rationalize Denominators

i) $\dfrac{2}{\sqrt{5}}$ as $\dfrac{\text{number}}{\text{something w/ out radial}}$

$$\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{(\sqrt{5})^2} = \frac{2\sqrt{5}}{5^{2/2}} = \frac{2\sqrt{5}}{5}$$

ii) $$\frac{3}{\sqrt[4]{x^5}} = \frac{3}{x^{4/5}} \cdot \frac{x^{1/5}}{x^{1/5}} = \frac{3x^{1/5}}{x^{\frac{4}{5}+\frac{1}{5}}} = \frac{3x^{1/5}}{x^1}$$

$$= \frac{3 \cdot (\sqrt[5]{x})}{x}$$

1.2

Section 1.3: Algebraic Expressions

Defn: A variable is a letter that denotes any real number

Defn: An alg. expression is a sum, power, root, div., etc of variables

↳ Ex: $2x^2 + x - 3$

$$\frac{x^2y^3}{\sqrt[5]{z}} - 24x + a^{7777}$$

$$\frac{wz + ab}{ab + xz}$$

Defn: A polynomial of degree n is an alg. expression of

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$$

where a_i = some reall #, x is our variable, $a_n \neq 0$.

↳ $7x + 3$, deg = 1

$8x^3 + 42x - 7$, deg = 3

$x^2 - 2x + 1$, deg = 2 (quadratic polynomials)

Rmk: We can add poly.

↳ $(3x^4 + x^2 + 7) + (7x^4 + x^3 - 77x + 8)$

$= 10x^4 + x^3 + x^2 - 77x + 15$

We can mult poly

↳ $(x-1)(x+1) = x^2 + x - x - 1 = x^2 - 1$

Rmk: Book has a lot of formulas ⇝ ignore these.

Defn: Undoing the prod of poly (or more gen alg expressions) is called factoring

↳ parts are called the factors

↳ Ex: $x^2 - 25 = \underbrace{(x+5)}_{\text{factors}}\underbrace{(x-5)}_{\text{factors}}$

Ex: $4x^2 - 8x = x(4x-8) = 4x(x-2)$

Ex: $8x^6y^3 + 6x^3y^3 + 8x^3 + 6$

$= y^3(8x^6 + 6x^3) + (8x^3+6)$

$= y^3x^3(8x^3+6) + (8x^3+6)\cdot 1$

$= (8x^3+6)(y^3x^3+1)$

Ex: Factoring Quads via Trial and Error

$x^2 - 7x + 10 = (x+s)(x+r)$.

want to solve for r, s.

$= x^2 + rx + sx + rs$

$= x^2 + (r+s)x + rs$

$\Rightarrow -7 = r+s, \quad 10 = rs$

By observation, $r = -2, \ s = -5$

$(x-2)(x-5) = x^2 - 2x - 5x + 10 = x^2 - 7x + 10$.

Ex: $x^2 - 2x + 1 = (x+s)(x+r) = (x-1)^2$

$\Rightarrow -2 = r+s, \quad 1 = rs$.

$\Rightarrow r = -1, \ s = -1$

$\frac{-2}{3} = -\frac{2}{3}$

Ex: Factor alg. exp. that look like quad. poly.

$\hookrightarrow 3x^{4/3} - 6x^{1/3} + 6x^{-2/3}$ } Idea, pull out power of x as lowest first factor.

$= x^{-2/3}(3x^{6/3} - 6x^1 + 6)$

$= x^{-2/3}(3x^2 - 6x + 6)$

$= 3x^{-2/3}(x^2 - 2x + 2)$

$=$ factor this... further.

$x^3 + x^2 + x$

$x(x^2 + x + 1)$