Lecture #19

Warm-up: Please compute:

1) $\cos^{-1}(\cos(7\pi/6)) = \cos^{-1}(-\sqrt{3}/2) = 5\pi/6.$

2) $\sin^{-1}(\cos(3\pi/4)) = \sin^{-1}(-\sqrt{2}/2) = -\pi/4.$

3) $\cos(\cos^{-1}(1/\pi)) = 1/\pi.$

Recall: $f^{-1}(x) = y$ st $f(y) = x$

$\cos^{-1}(x) = y$ st $\cos(y) = x$ and $0 \le y \le \pi$

$\sin^{-1}(x) = y$ st $\sin(y) = x$ and $-\pi/2 \le y \le \pi/2.$

Remark: 1) $\cos(\cos^{-1}(x)) = x$

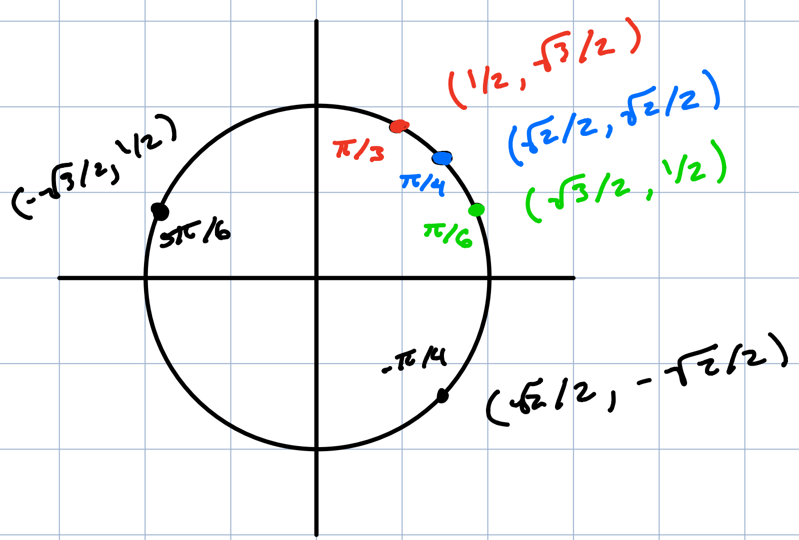
$\sin(\sin^{-1}(x)) = x$

2) $\cos^{-1}(\cos(x)) = x$ is not nec. true.

$\sin^{-1}(\sin(x)) = x$ " " " "

$\hookrightarrow$ $\cos^{-1}(x) = y$ st $\cos(y) = x$ and $0 \le y \le \pi.$

$\Rightarrow \cos(\cos^{-1}(x)) = \cos(y) = x.$

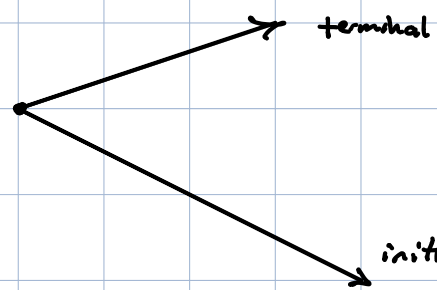


$\cos(7\pi/6)$

Section 6.1:

Defn: An angle is two rays (initial ray and a term ray) that both start at same point

↳ Ex:



The measure of an angle is an amount of degrees needed to rotate the initial ray into the terminal ray.

↳ clockwise rotate = neg. angle measurement

↳ counterclockwise rotation = pos. " "

Remark: • A degree is $1/360^{th}$ of the way around a circle.

• A 2π radian is the length of the path around the circle w/ circum 2π

• 360 degrees = 2π radians.

Ex: 1) Express $5\pi/6$ radians in degrees

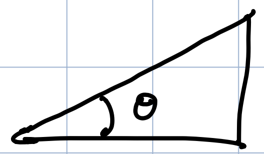
$$\frac{5\cancel{\pi}}{6}\ \cancel{\text{rad.}} \cdot \frac{360 \text{ deg.}}{2\cancel{\pi}\ \cancel{\text{rad.}}} = 5 \cdot \frac{60}{2} \text{ deg} = 150^\circ.$$

2) Express 56° in radians.

$$\frac{56\ \cancel{\text{deg.}}}{1} \cdot \frac{2\pi \text{ rad}}{360\ \cancel{\text{deg.}}} = \frac{112\pi}{360} \text{ rad}.$$

Section 6.2: Trig of Right Δs.

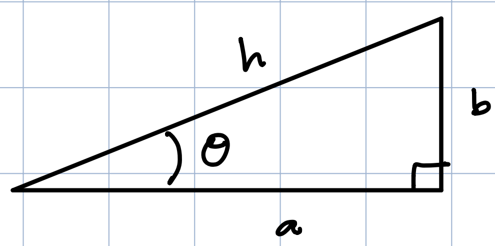
Rmk: Given a Δ,



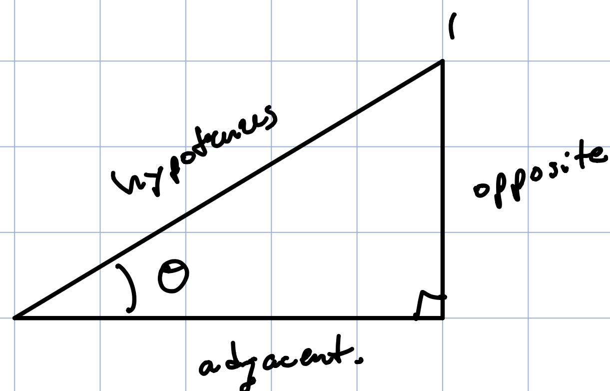
then two sides give two rays and thus an angle.

Defn: A right triangle is a triangle w/ one angle equal to $90^\circ = \pi/2$ rad

↳ Ex:



Defn: Let θ be an acute angle (less than 90°) formed by 2 sides of a right Δ.



Then the trig ratios associated to θ are:

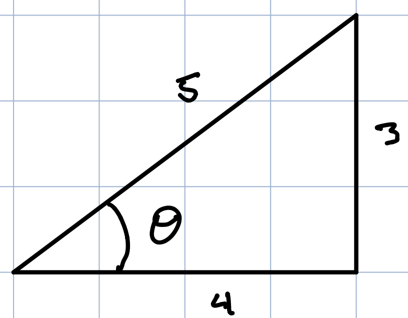
$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}}$$

$\csc(\theta) = \dfrac{hyp}{opp}$ $\qquad$ $\sec(\theta) = \dfrac{hyp}{adj}$

$\tan(\theta) = \dfrac{opp}{adj}$ $\qquad$ $\cot(\theta) = \dfrac{adj}{opp}$.

$\hookrightarrow$ SOHCAHTOA.

Ex:



$\sin(\theta) = \dfrac{3}{5}$

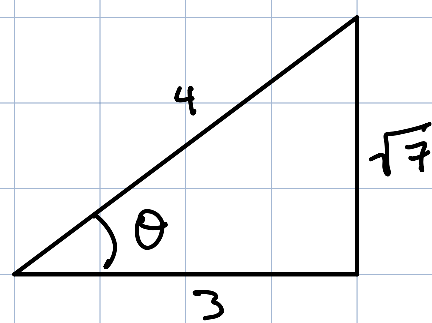
$\cos(\theta) = \dfrac{4}{5}$

$\tan(\theta) = \dfrac{3}{4}$.

Ex: If $\cos(\theta) = \dfrac{3}{4}$, then what are the other trig ratios associated to the angle θ.

$\hookrightarrow$ $\tan(\theta)$, $\sin(\theta)$, $\sec(\theta)$, $\csc(\theta)$, $\cot(\theta)$.

$$\tan(\theta) = \frac{\sqrt{7}}{3}, \quad \sin(\theta) = \frac{\sqrt{7}}{4}, \quad \sec(\theta) = \frac{4}{3}, \quad \csc(\theta) = \frac{4}{\sqrt{7}}, \quad \cot(\theta) = \frac{3}{\sqrt{7}}.$$

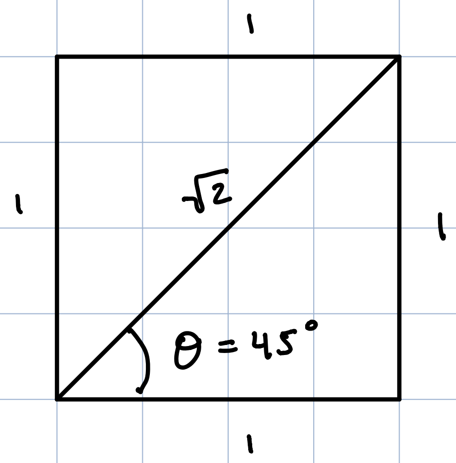


$hyp^2 = adj^2 + opp^2$

$\hookrightarrow$ $16 = 9 + (opp)^2 \Rightarrow 7 = (opp.)^2$.

$\Rightarrow opp = +\sqrt{7}$.

Ex:



1) $\sin(45°) = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

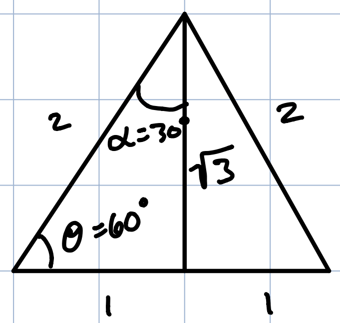
2) $\cos(45°) = \sqrt{2}/2$.

3) $\frac{45 \cancel{\text{deg}}}{1} \cdot \frac{2\pi \text{ rad}}{360 \cancel{\text{deg}}} = \frac{90\pi}{360} \text{ rad}$

$= \pi/4$

$\Rightarrow \sin(\pi/4) = \sqrt{2}/2 = \cos(\pi/4)$

Ex:



1) $\sin(60°) = \sqrt{3}/2$

2) $\cos(60°) = 1/2$

3) $\frac{60 \text{ deg}}{1} \cdot \frac{\pi \text{ rad.}}{180 \text{ deg}} = \frac{\pi}{3} \text{ rad.}$

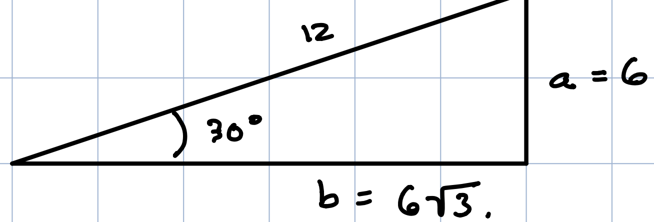
$\Rightarrow \sin(\pi/3) = \sqrt{3}/2$

$\cos(\pi/3) = 1/2$.

$\hookrightarrow \sin(\pi/6), \cos(\pi/6)$.

$$\boxed{1 = \frac{2\pi \text{ rad.}}{360 \text{ deg.}} = \frac{\pi \text{ rad}}{180 \text{ deg}}}$$

Ex:



What are a and b?

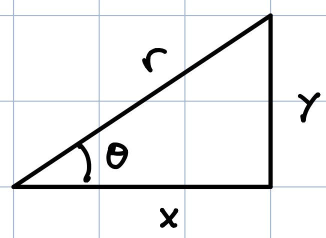
$\hookrightarrow \sin(30°) = a/12 \Rightarrow 12 \sin(30°) = a$

$\Rightarrow a = 12\left(\frac{1}{2}\right) = 6$

$\hookrightarrow \cos(30°) = b/12 \Rightarrow 12 \cos(30°) = b$

$$\Rightarrow \quad b = 12\left(\sqrt{3}/2\right) = 6\sqrt{3}$$

Fact:



$$\Rightarrow \quad r\cos(\theta) = x$$
$$r\sin(\theta) = y$$
$$r^2 = x^2 + y^2$$

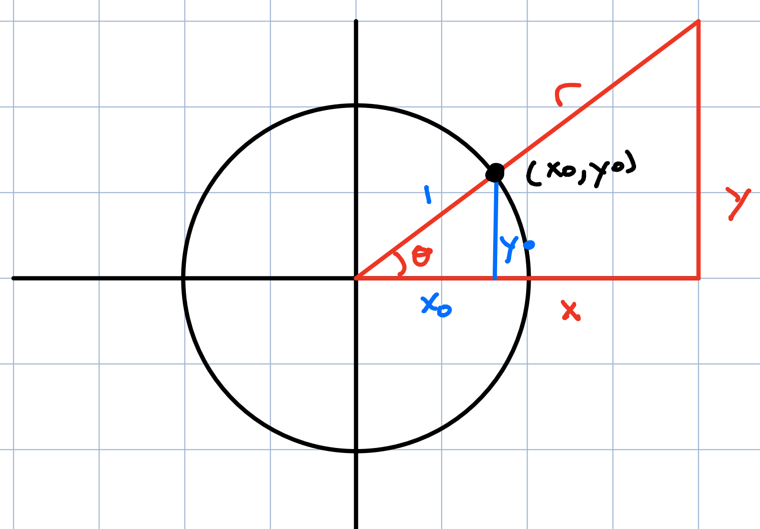
Cor: $\sin^2\theta + \cos^2\theta = 1$

Proof: $\dfrac{y^2}{r^2} + \dfrac{x^2}{r^2} = \dfrac{y^2 + x^2}{r^2} = \dfrac{r^2}{r^2} = 1$

Rmk: $\sin^2\theta + \cos^2\theta = 1$ holds the information of pathagorians thrm.

Section: 6.3

Rmk: Unit circle approach is equiv. to the right Δ approach.



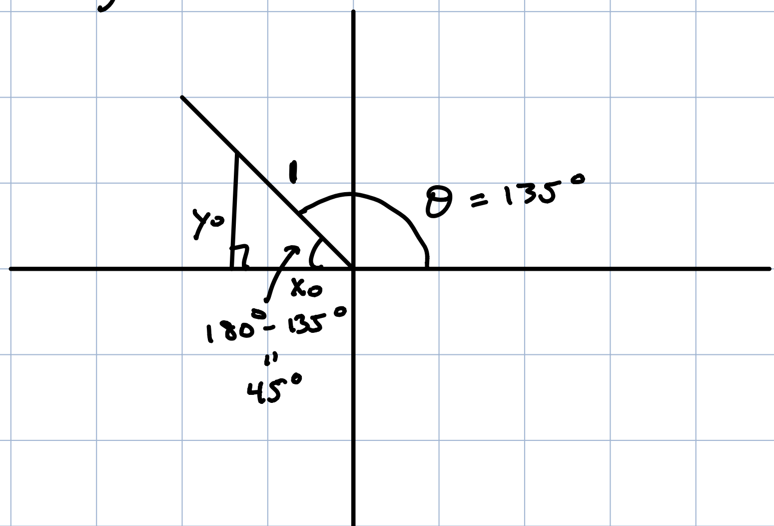
Unit circle approach $\Rightarrow$ $\cos(\theta) = x_0$

$\sin(\theta) = y_0$

Right Δ " $\Rightarrow$ $\cos(\theta) = x_0/1 = x_0$

$\sin(\theta) = y_0$.

Ex: a) $\cos(135°)$



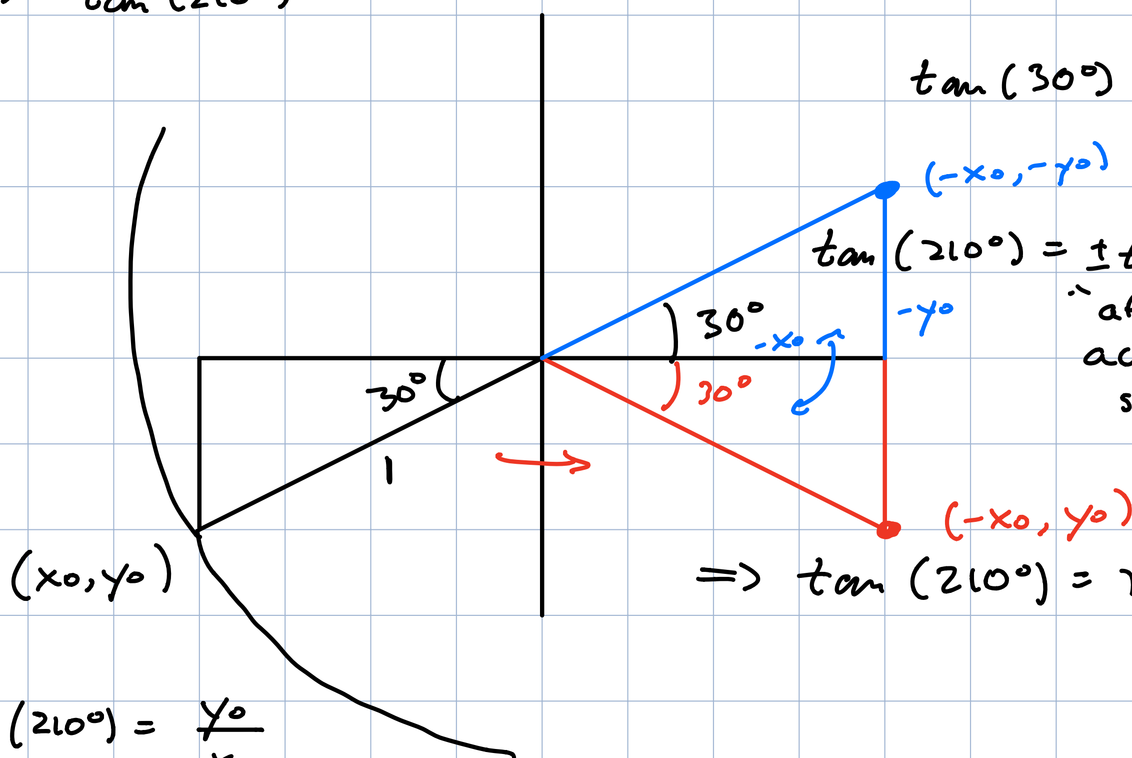
$\Rightarrow$ $\cos(45°) = x_0 = \sqrt{2}/2$

$\Rightarrow$ $\cos(135°) = -\sqrt{2}/2$.

$7\pi/6$

b) $\tan(210°)$

$\tan(30°) = \sqrt{3}/3$.



$\tan(210°) = \pm\tan(30°)$ "after we account for sign."

$\Rightarrow$ $\tan(210°) = \sqrt{3}/3$.

$\tan(210°) = \dfrac{y_0}{x_0}$

$= \sin(210°)/\cos(210°)$.

$\Rightarrow \tan(30°) = -y_0/-x_0 = y_0/x_0 = \tan(210°).$

$\tan(7\pi/6)$ is computed via
term pt $7\pi/6$ = term. pt of $\pi/6$ but w/
negs on both x, y - coords.