Lecture # 18

Warm-up: i) Graph the function $f(x) = \tan(x/2 + \pi/4)$.

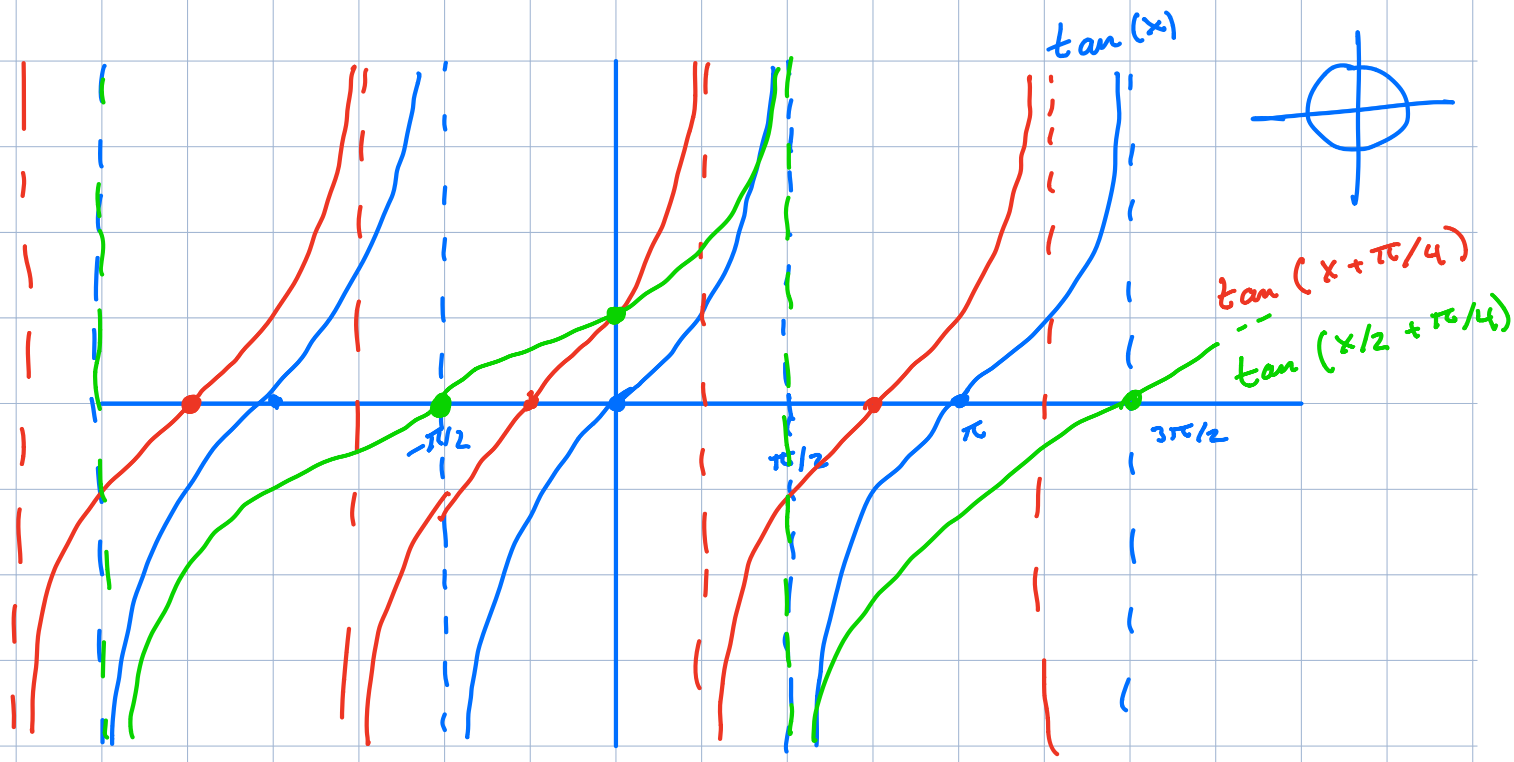
What is its period?

↪ The period of a fcn f is a value T st $f(t+T) = f(t)$.

↪ sin, cos period 2π

↪ tan period π.

$\tan = \dfrac{\sin(t)}{\cos(t)}$



Period $= 2\pi$

↪ $\tan(Kx+b)$ has period π/K.

↪ $\sin(Kx+b)$ " " $2\pi/K$.

$\cos(Kx+b)$ " " $2\pi/K$.

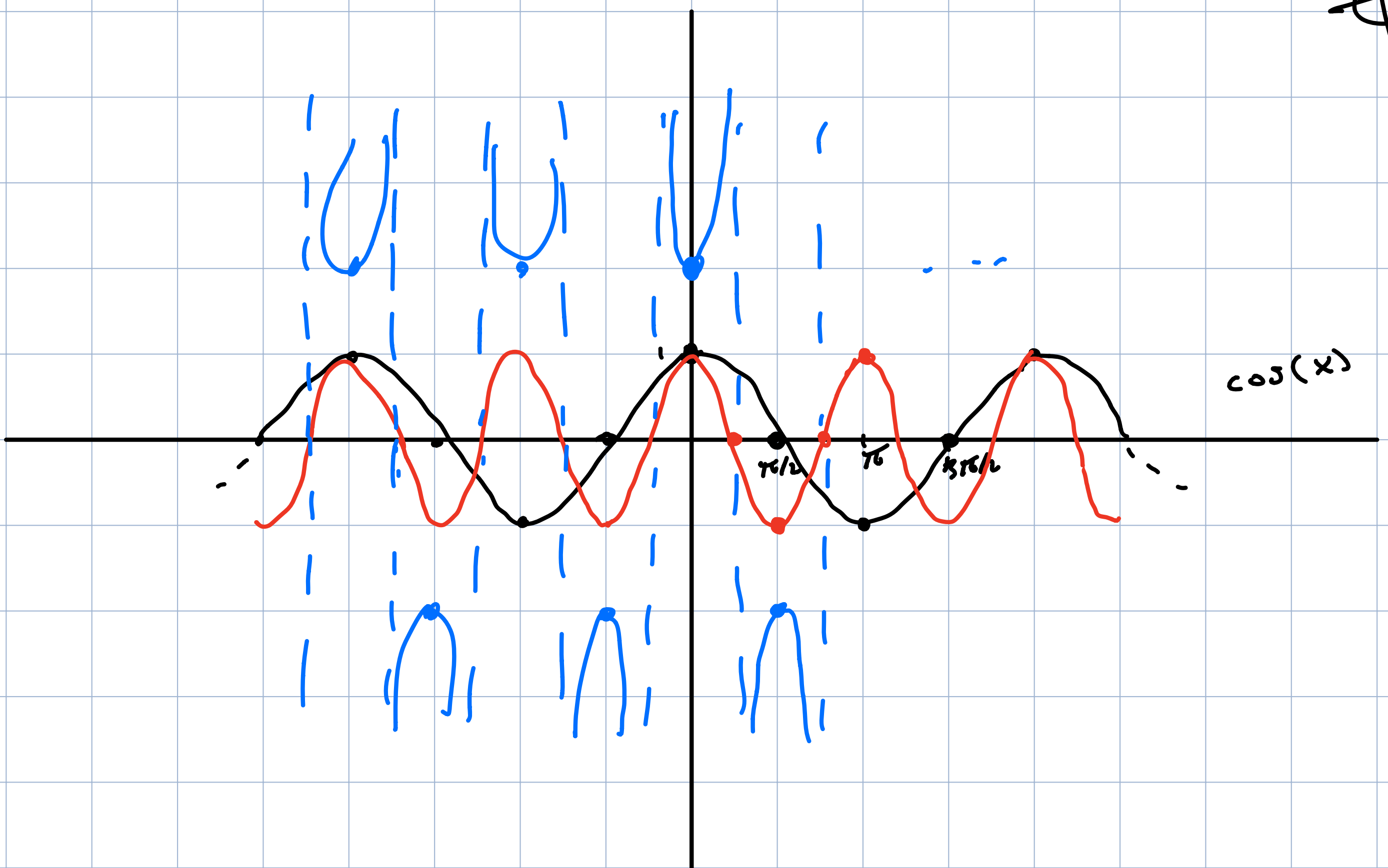
ii) Graph the function $f(x) = 2\sec(-2x)$

What is its period?

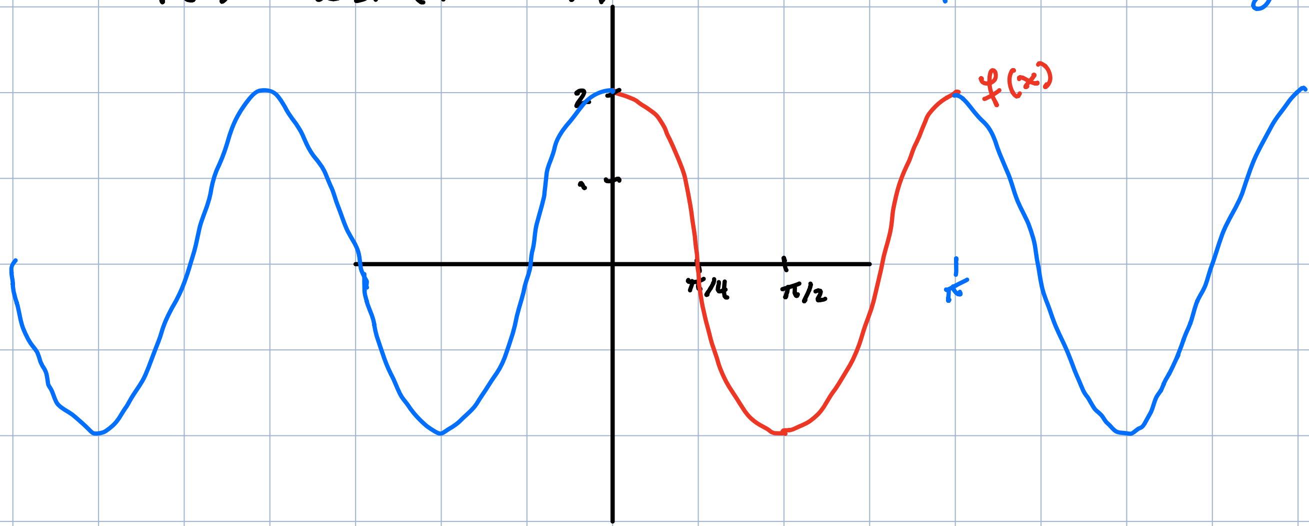
$\sec = \frac{1}{\cos}$.

$\Leftrightarrow \quad 2\sec(-2x) = \dfrac{2}{\cos(-2x)} = \dfrac{2}{\cos(2x)}$

$\Leftrightarrow \quad \cos = \text{even}, \quad \sin = \text{odd}, \quad \tan = \text{odd}.$



iii) The following is one period of a fcn of the form
$f(x) = a\sin(Kx+b)$. ⟶ one repeated chunk of graph

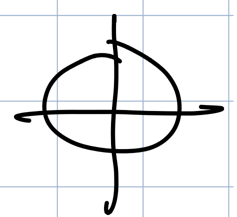


What are possible values of a, b, and K.

If $f(x) = a\cos(Kx+b)$, then what are a, K, b

↳ Amplitude of f, ie, max height $= 2$

$\Rightarrow \quad a = 2$



↳ Period is $2\pi/K = \pi$ (π by graph)

$\Rightarrow \quad K = 2$

↳ b we test a value

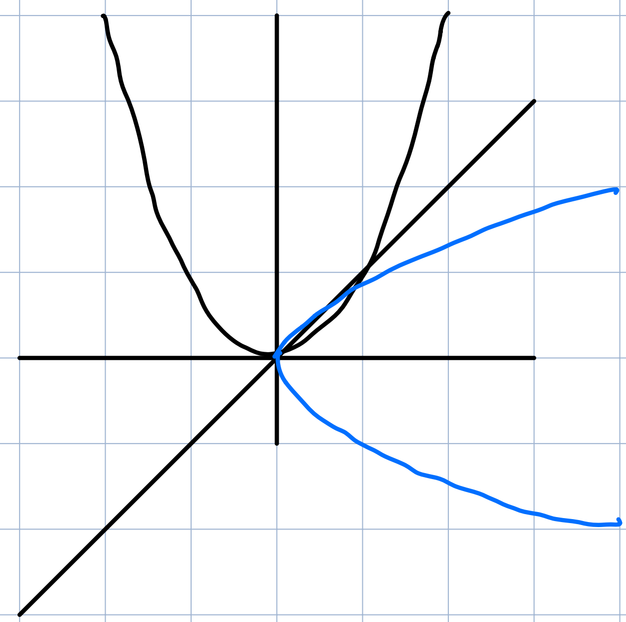
$f(0) = 2\sin(2(0)+b) = 2$

$\Rightarrow \quad \sin(b) = 1$

$\Rightarrow \quad b = \pi/2$.

Section 5.5: Inverse Trig Functions.

Ex: $f(x) = x^2$



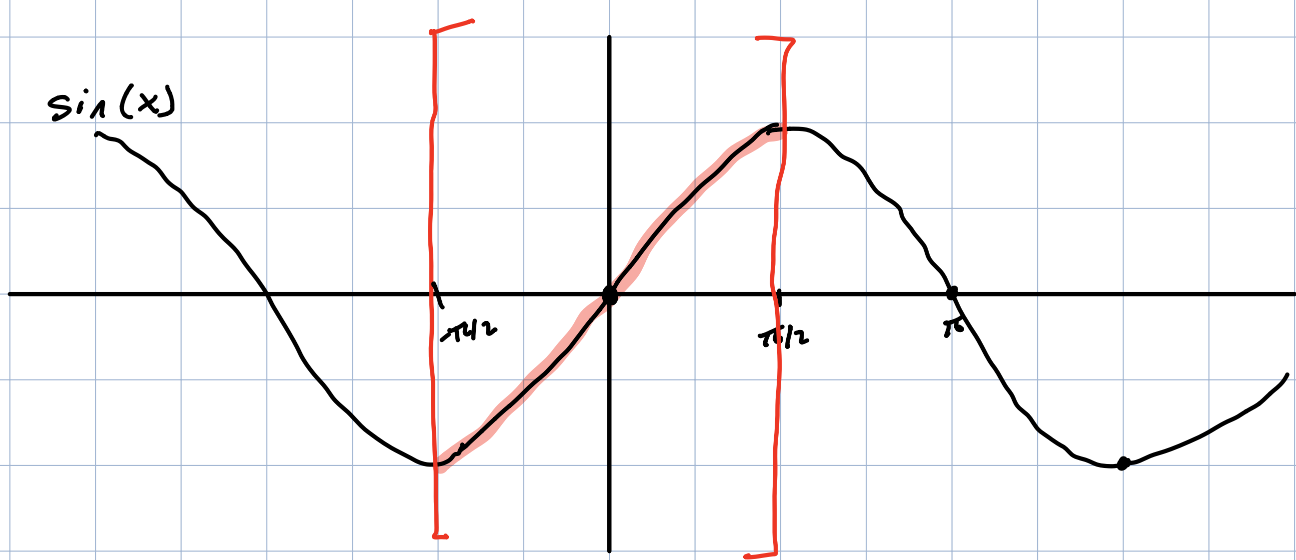
$\hookrightarrow \sqrt{x} = y$ st $y^2 = x$ when $x \geq 0$.

$x \geq 0$, $(\sqrt{x})^2 = x$, $\sqrt{x^2} = x$.

$\Rightarrow$ for $x \geq 0$, x^2 has an inverse, $f^{-1}(x) = \sqrt{x}$.

$\hookrightarrow$ we restricted the domain so that fcn restricted to this domain satisfies the horizontal line test.

Rmk: $\sin(x)$



satisfies the horizontal line test

Defn: $\sin^{-1}(x) = y$ st $\sin(y) = x$ for $-\pi/2 \le y \le \pi/2$

$-1 \le x \le 1$

$\Rightarrow$ $\text{dom}(\sin^{-1}) = [-1, 1] = \text{range}(\sin)$

$\text{range}(\sin^{-1}) = [-\pi/2, \pi/2] = \text{dom}(\sin$ restricted$)$.

Ex: $\sin^{-1}(1/2) = y$ st $\sin(y) = \frac{1}{2}$ <u>and</u> $-\pi/2 \le y \le \pi/2$.

$\Rightarrow y = \pi/6$.

$\sin^{-1}(-\sqrt{2}/2) = t$ st $\sin(t) = -\sqrt{2}/2$ <u>and</u> $-\pi/2 \le t \le \frac{\pi}{2}$

$\Rightarrow y = -\pi/4$

$\sin^{-1}(\sqrt{3}/2) = t$ st $\sin(t) = \sqrt{3}/2$ <u>and</u> $-\pi/2 \le t \le \frac{\pi}{2}$.

$t = \pi/3$

Ex: $\sin^{-1}(\sin(2\pi/3)) = \sin^{-1}(\sqrt{3}/2) = t$

st $\sin(t) = \sqrt{3}/2$ and $-\pi/2 \le t \le \pi/2$

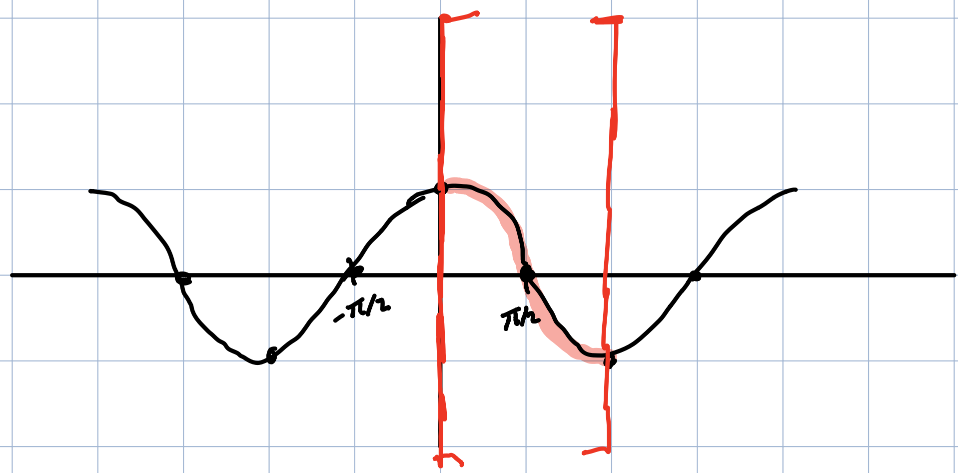
$\Rightarrow t = \pi/3$

So $\sin^{-1}(\sin(2\pi/3)) = \pi/3$.

$f^{-1}(x) = \sqrt{x}$, $f(x) = x^2$

$f^{-1}(f(-2)) = \sqrt{(-2)^2} = \sqrt{2^2} = \sqrt{4} = 2$.

Rmk:



Between $[0, \pi]$ cos is 1-to-1.

Defn: $\cos^{-1}(x) = t$ st $\cos(t) = x$ and $0 \le t \le \pi$

$\Rightarrow$ $\text{dom}(\cos^{-1}) = [-1, 1] = \text{range}(\cos)$

$\text{range}(\cos^{-1}) = [0, \pi] =$ dom. of restriction.

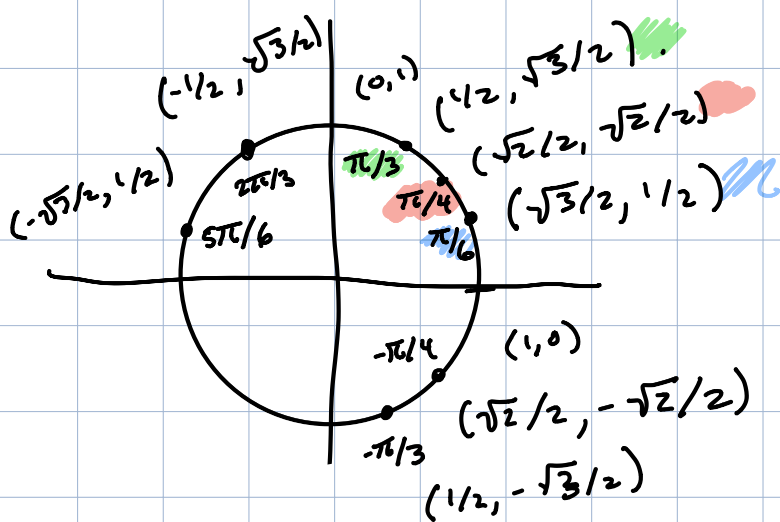
Ex: • $\cos^{-1}(-\sqrt{3}/2) = t$ st $\cos(t) = -\sqrt{3}/2$, $0 \le t \le \pi$

$\Rightarrow t = 5\pi/6$

• $\cos^{-1}(\cos(5\pi/3)) = \cos^{-1}(1/2) = t$

st $\cos(t) = 1/2$ and $0 \le t \le \pi$ $\qquad \frac{5\pi}{3} = \frac{2\pi}{3} + \frac{3\pi}{3}$

$\Rightarrow t = \pi/3$



Def: $\tan^{-1}(x) = t$ st $\tan(t) = x$ and $-\pi/2 < t < \pi/2$.

$\Leftrightarrow$ $\text{dom}(\tan^{-1}(x)) = \mathbb{R} = \text{range}(\tan)$

$\text{range}(\tan^{-1}(x)) = (-\pi/2, \pi/2)$

Ex: $\tan^{-1}(\sqrt{3}) = t$ st $\tan(y) = \sqrt{3}$, $\frac{-\pi}{2} < t < \frac{\pi}{2}$

$t = \pi/3$