Lecture # 17

Correction: For the logarithm rules:

i) $\log_a(A \cdot B) = \log_a(A) + \log_a(B)$

ii) $\log_a(A/B) = \log_a(A) - \log_a(B)$

iii) $\log_a(A^C) = C \cdot \log_a(A)$

We assume $A, B, C > 0$ so the RHS's make sense.

Warm-up: What is

neg. values => clockwise
pos. values => counter clockwise.

i) $\cot(-32\pi/6) =$

$$\underbrace{-32\pi/6} = -5\pi - \frac{\pi}{3} = -4\pi - \pi - \frac{\pi}{3}$$

term pt of $(\sim) =$ term pt of $2\pi/3$

$= (-1/2, \sqrt{3}/2)$

$$\Rightarrow \frac{\cos(-32\pi/6)}{\sin(-32\pi/6)} = \frac{-1/2}{\sqrt{3}/2} = \frac{-1}{\sqrt{3}}$$

ii) $\sec(-5\pi/6) = 1/\cos(-5\pi/6) = -2/\sqrt{3}$

iii) $\sin(-9\pi/4) =$

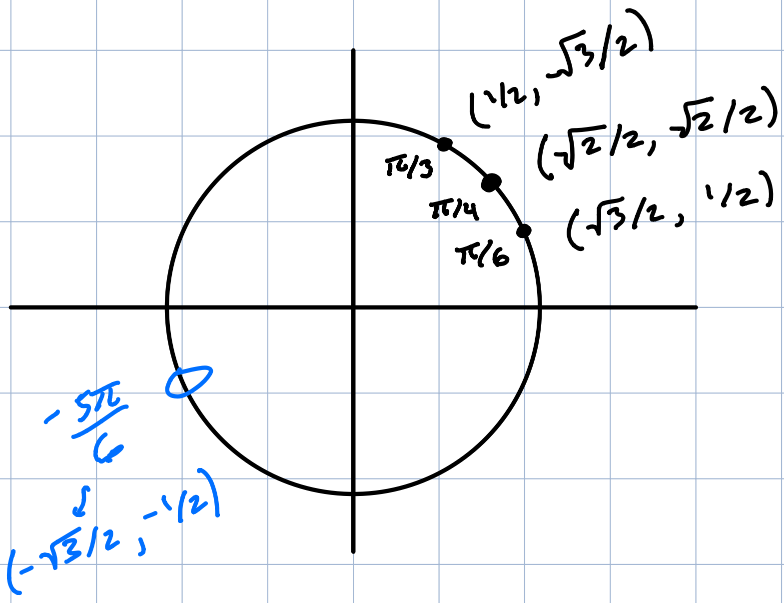
$$\underbrace{-9\pi/4} = -2\pi - \frac{\pi}{4}$$

$\Rightarrow$ term pt of $(\sim) =$ term pt of $-\pi/4$

$= (\sqrt{2}/2, -\sqrt{2}/2)$

$\Rightarrow \sin(-9\pi/4) = -\sqrt{2}/2$.

Please use the unit circle and values below



$\pi/2$

$3\pi/2$

$5\pi/2$

Rmks

Fcn	Dom	Range	even/odd
sin	$\mathbb{R}$	$[-1, 1]$	odd
cos	$\mathbb{R}$	$[-1, 1]$	even.
tan	$t \neq$ odd mult of $\pi/2$	$\mathbb{R}$	odd
csc	$x \neq$ mult of π	$\mathbb{R}$	odd
sec	$t \neq$ odd mult of $\pi/2$	$\mathbb{R}$	even
cot	$x \neq$ mult of π.	$\mathbb{R}$	odd

$-\pi/2, -3\pi/2$
$-5\pi/2, \pi/2,$
$3\pi/2, 5\pi/2$
$7\pi/2, \ldots$ etc.

f even
g odd
$\Rightarrow f \cdot g$ odd.

- $\tan = \sin / \cos$

  $\text{dom}(\tan) = \{ t \mid \cos(t) \neq 0 \}$.

- $\sec = 1/\cos$

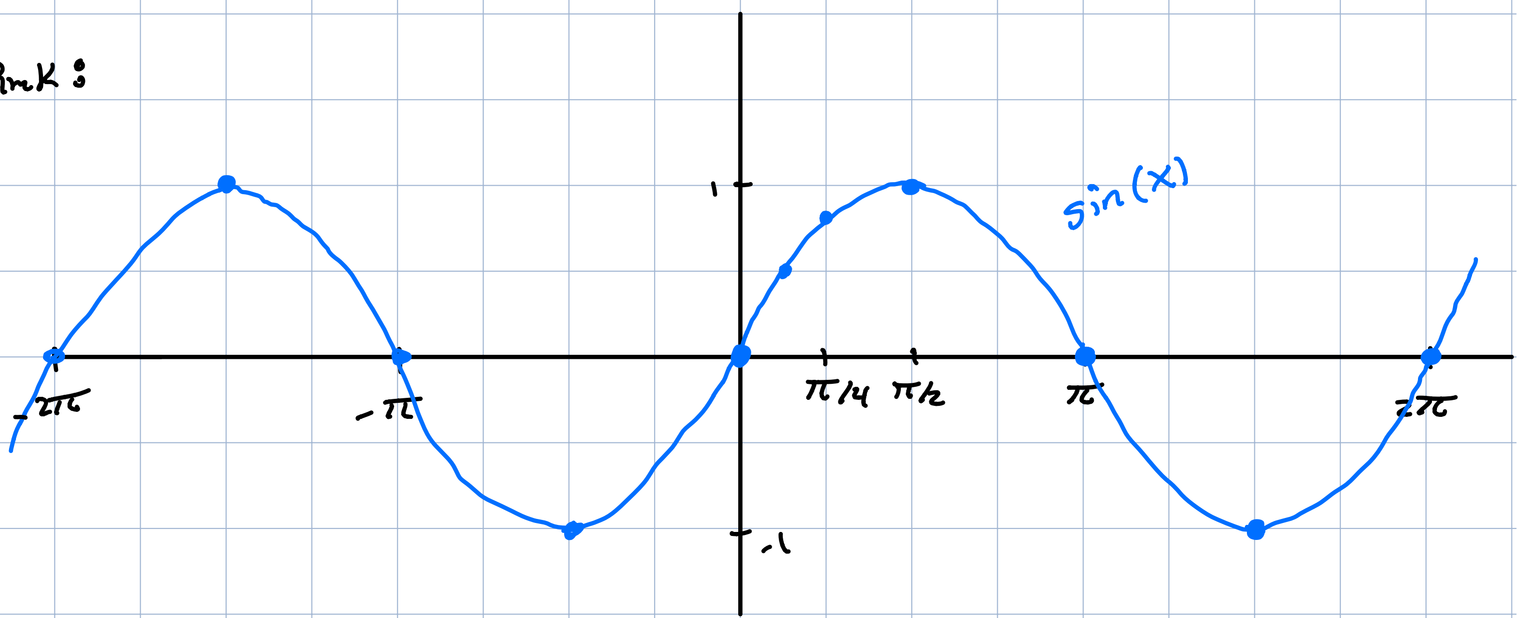
Section 5.3: Trig. Graphs

Remark:
- $\sin(t + 2\pi) = \sin(t)$
- $\cos(t + 2\pi) = \cos(t)$

} sin, cos have period 2π.

Defn: A fcn f is periodic w/ period T if
$$f(t+T) = f(t)$$

Rmk:



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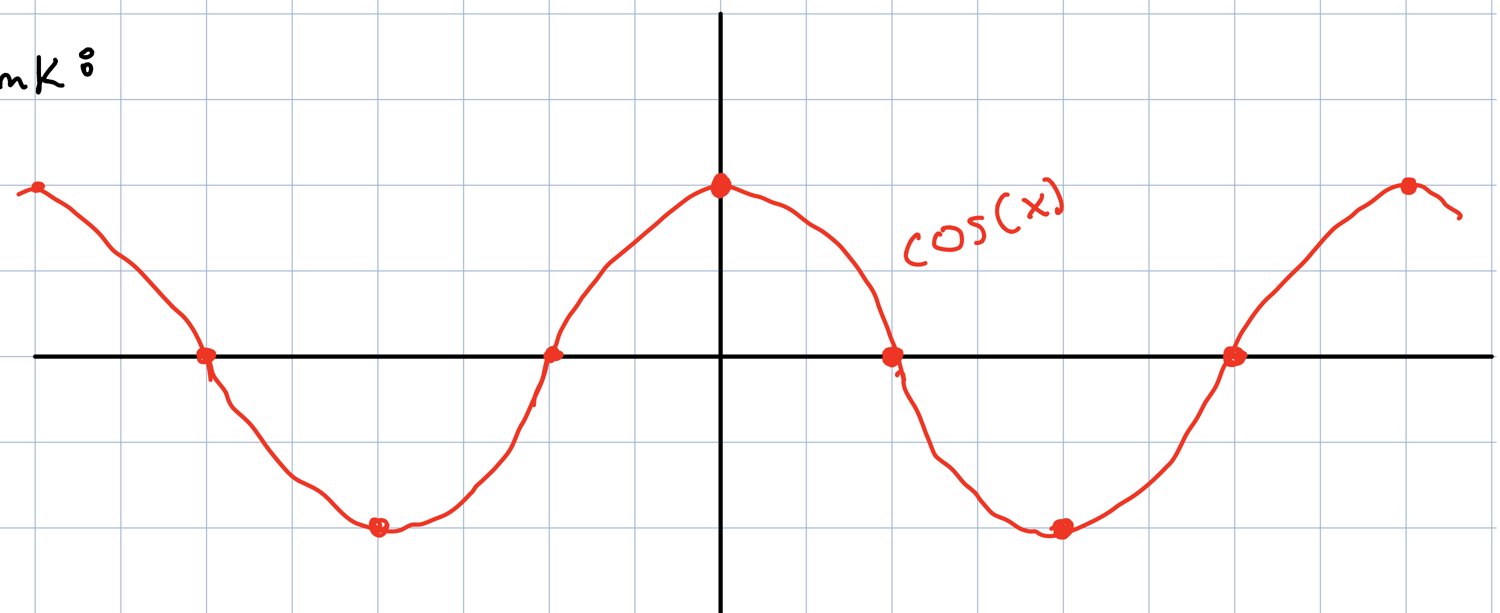
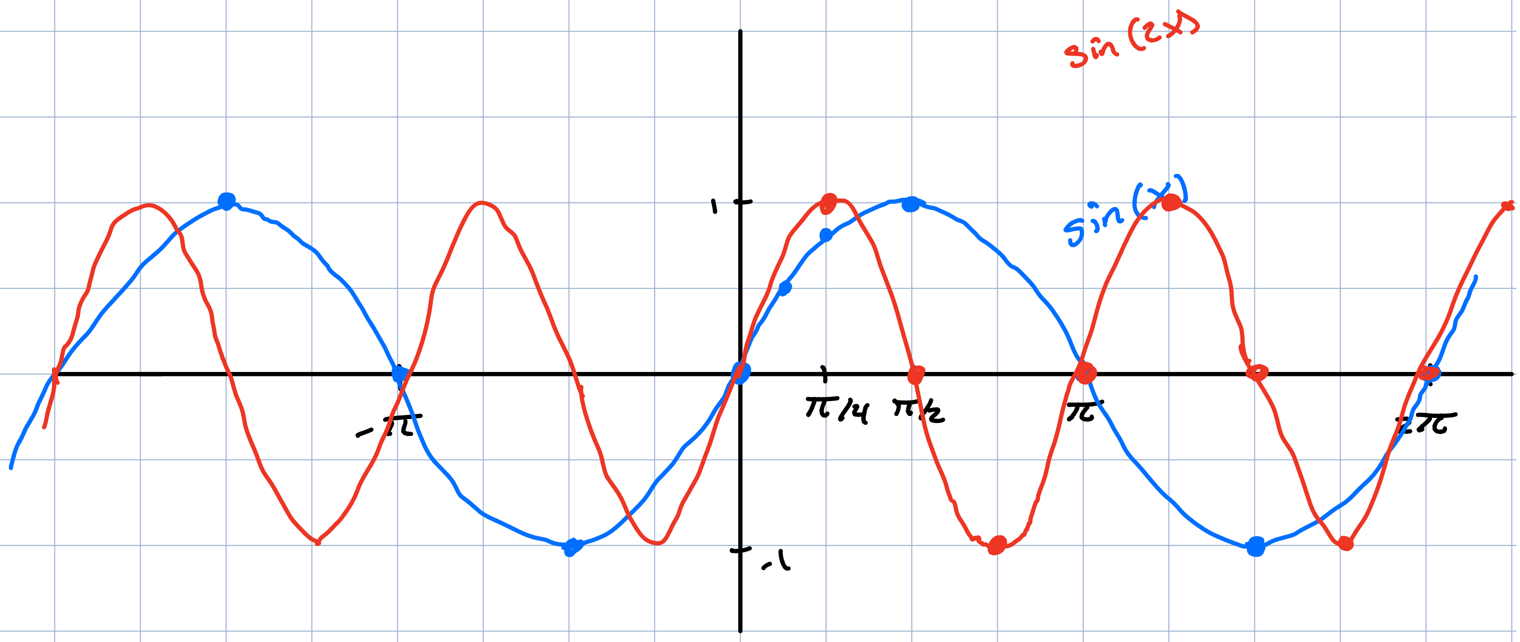
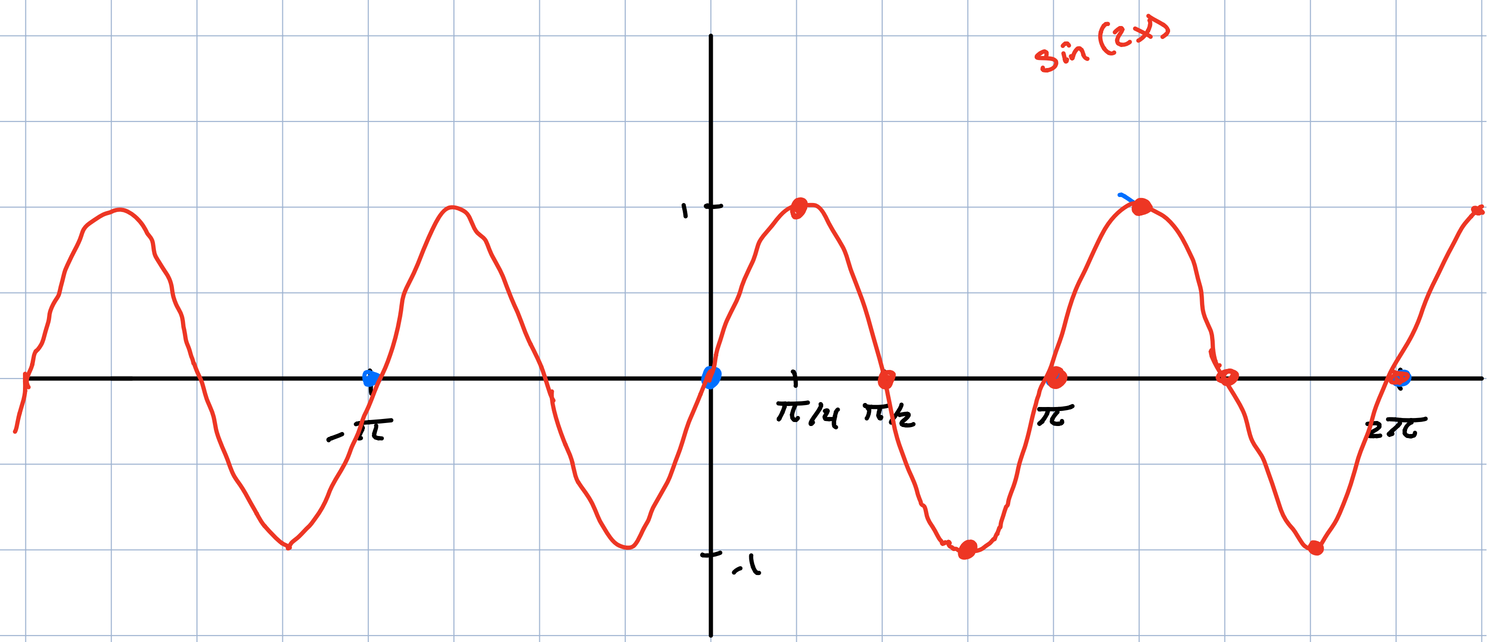
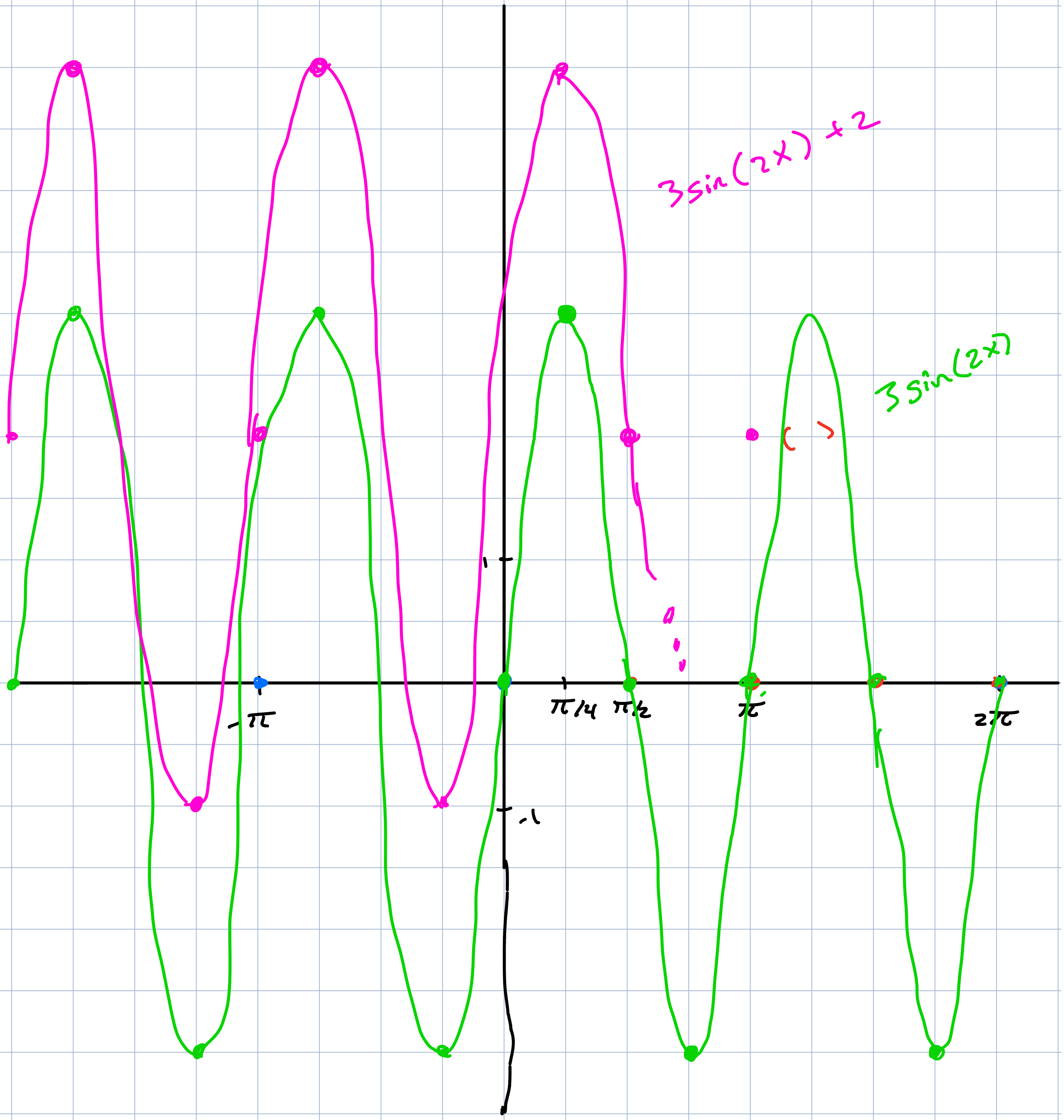


Ex: $f(x) = 3\sin(2x) + 2$







3sin(2x) + 2
3sin(2x)
1
-1
-π
π/4
π/2
π
2π

Defn: The amplitude of $f(x) = a \cdot \sin(x)$ is $|a|$

" " " " $= b \cdot \cos(x)$ " $|b|$.

Fact: The fcn $f(x) = a \cdot \sin(K \cdot x - b)$ has

amplitude $= |a|$

period $= 2\pi / K$

and its graph repeats over $[b, \frac{2\pi}{K} + b]$

(sim. for $f = b \cdot \cos(Kx - b)$.

$\hookrightarrow \sin(K(x + \frac{2\pi}{K})) = \sin(Kx + 2\pi)$

$= \sin(Kx)$

Ex: • $3\cos(x/4 - \pi/4)$

$K = \frac{1}{4}$

$\hookrightarrow$ Amp. $= 3$

periodic $= 2\pi/K = 2\pi/\frac{1}{4} = 8\pi$

repeats over $[\pi/4, 8\pi + \pi/4]$

Section 5.4: More Trig Graphs.

Fact: i) $\tan(t + \pi) = \tan(t)$

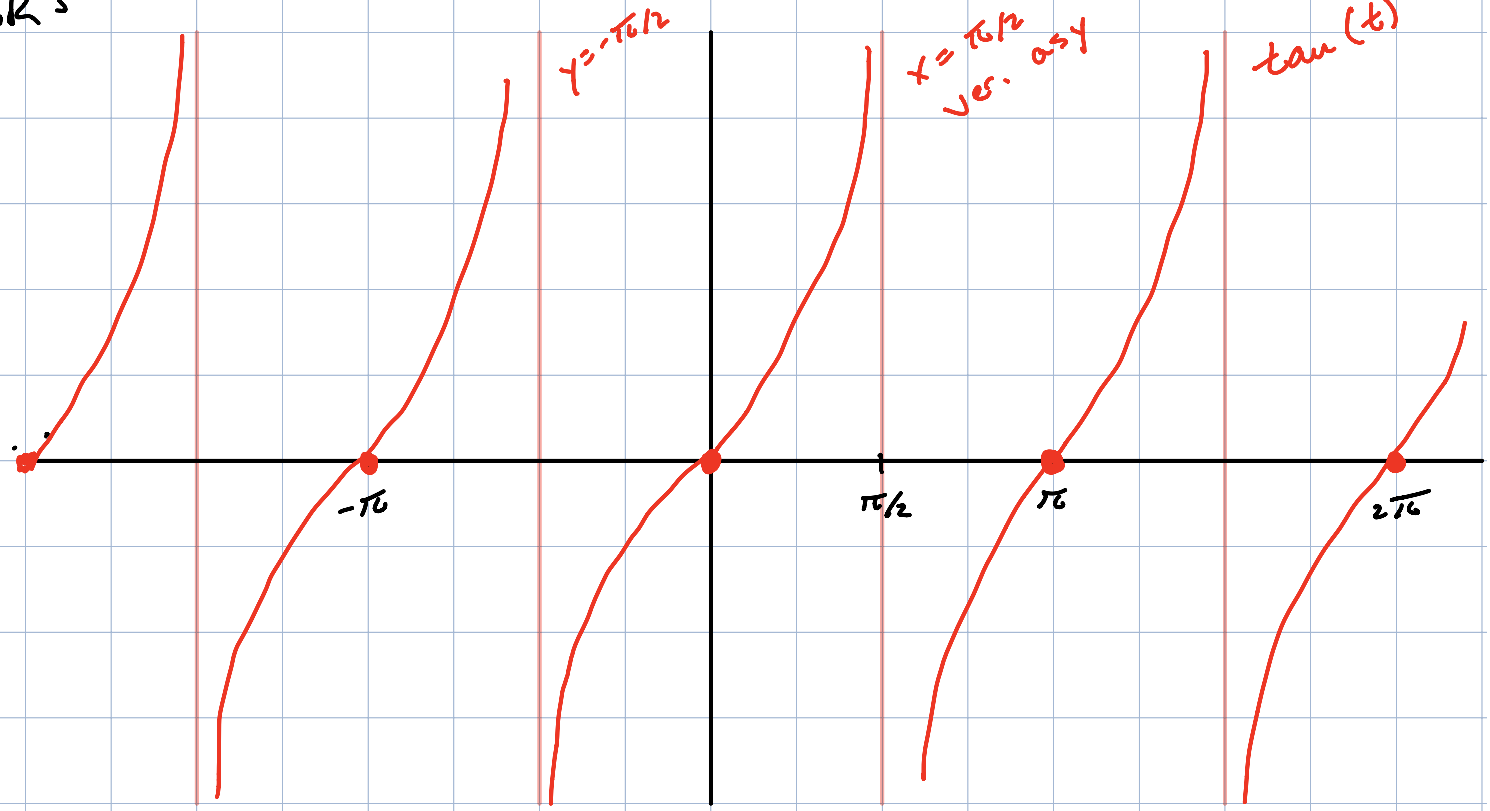
$\csc(t) = \frac{1}{\sin(t)}$

ii) $\cot(t + \pi) = \cot(t)$

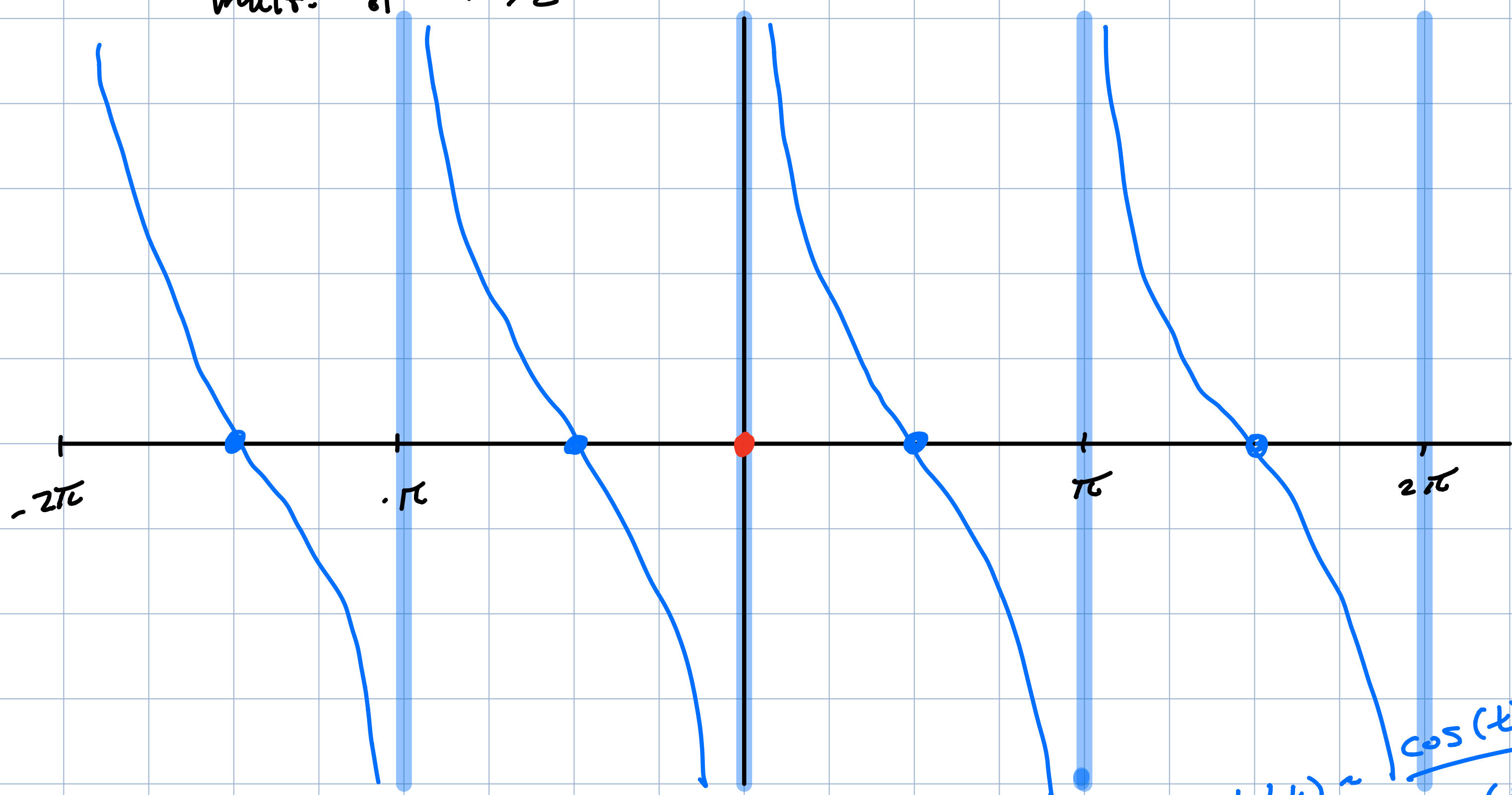
iii) $\csc(t + 2\pi) = \csc(t)$

iv) $\sec(t + 2\pi) = \sec(t)$.

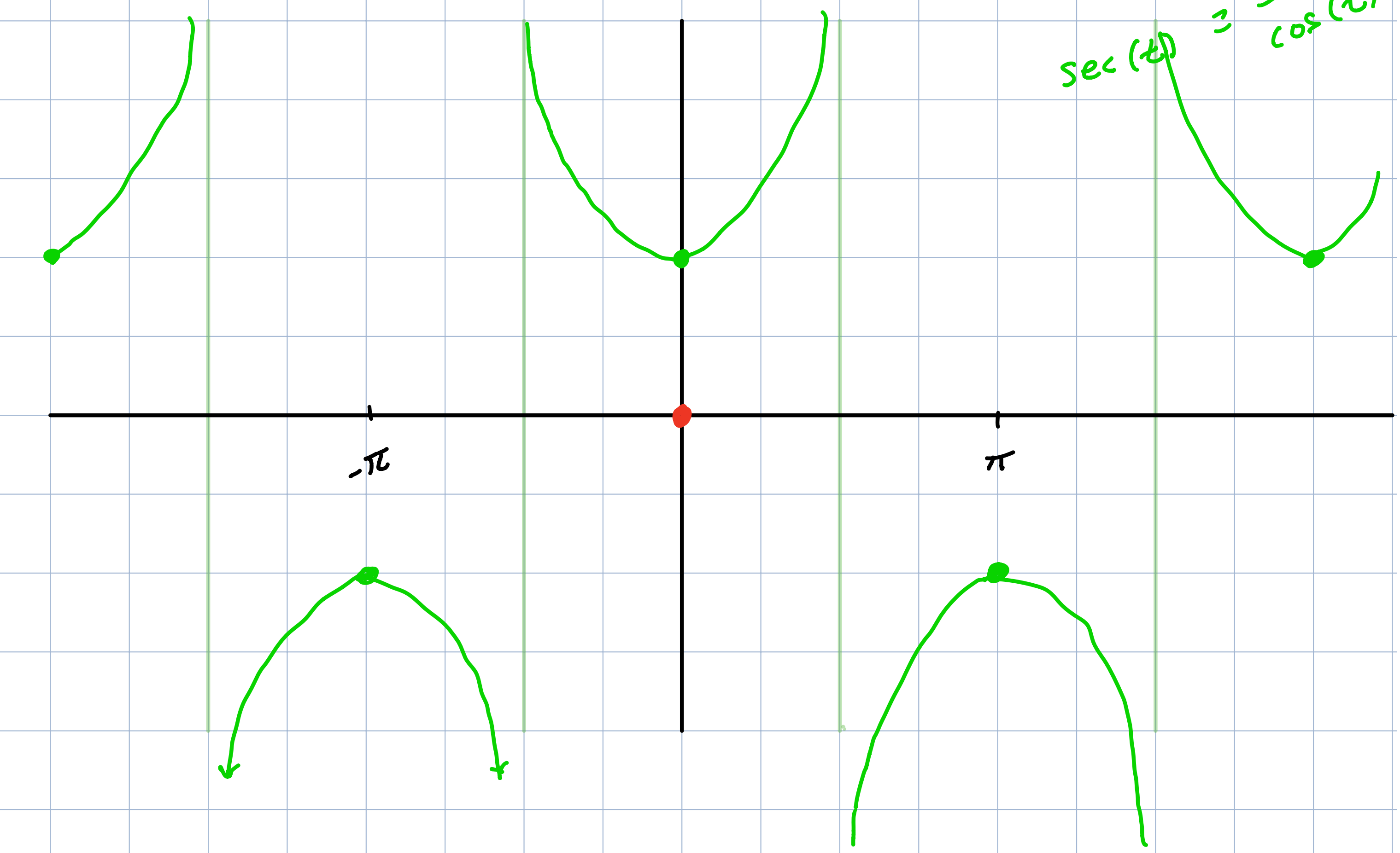
Rmk:



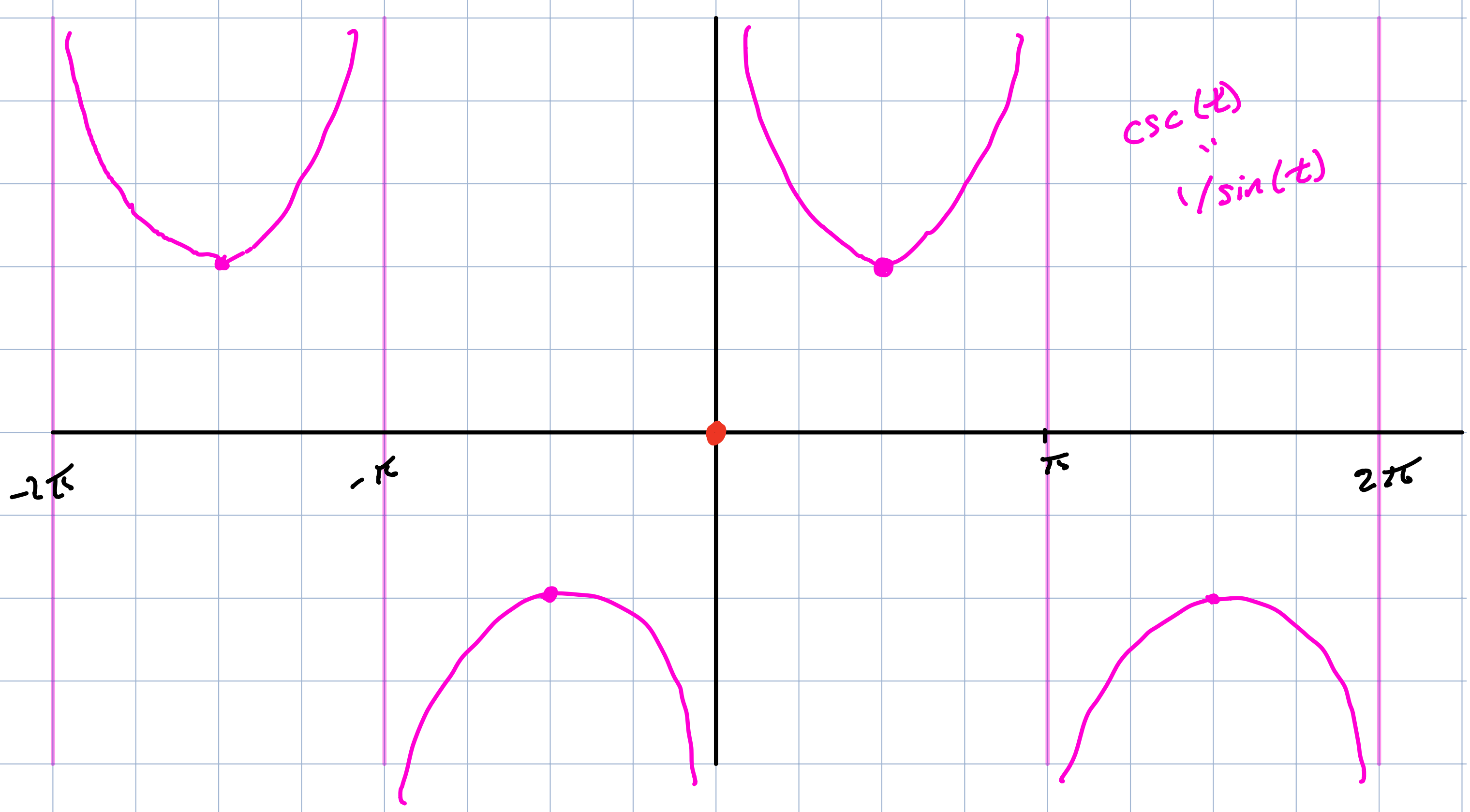
tan(t) has a ver. asy along x = every odd mult. of π/2



$\cot(t) = \dfrac{\cos(t)}{\sin(t)}$



$sec(t) = \frac{1}{cos(t)}$
$-\pi$
π



$csc(t) = 1/sin(t)$
-2π
$-\pi$
π
2π

Fact: i) $a \cdot \tan(Kx)$

↳ has period π/K

↳ ver. asy when $x =$ odd mult. of $\pi/2K$

ii) $a\cot(Kx)$

↳ " " π/K

↳ " " " " $=$ mult of π/K.

Ex: $f(x) = \tan\left(2(x - \pi/4)\right)$, then

this has period $\pi/2$