Lecture # 16

Warm-up: Find all real solutions

i) $e^{6x} - 2e^{3x} + 1 = 0$

(Realize the eqn as quad.)

$\leadsto x^6 - x^3 + 7 \qquad x^3 = y$

$y^2 - y + 7$

$y = e^{3x}$

$e^{6x} = (e^{3x})^2 = y^2$

$\Rightarrow y^2 - 2y + 1 = 0$

$\Rightarrow (y-1)^2 = 0$

$\log_a(1) = 0$

$\Rightarrow y = 1$

$\Rightarrow e^{3x} = 1$

$\Rightarrow 3x = \ln(e^{3x}) = \ln(1) = 0$

$\Rightarrow x = 0$

ii) $xe^{2x} + x^2e^{2x} = 0$

(Factor and solve)

$x^4 + x^3 + x^2 = 0$

$x^2(x^2 + x + 1) = 0$

$e^{2x}(x + x^2) = 0$

$e^{2x} \cdot x \cdot (1 + x) = 0$

$x = -1, 0$

a^x has range $(0, \infty)$

iii) $\log(x-2) + \log(x-1) = \log(x-3)$

$\Rightarrow \log((x-2)(x-1)) - \log(x-3) = 0$

$\Rightarrow \log\left(\frac{(x-2)(x-1)}{(x-3)}\right) = 0$

$\boxed{\log = \log_{10}}$

$\Rightarrow 10^{\log\left(\frac{(x-2)(x-1)}{x-3}\right)} = 10^0 = 1$

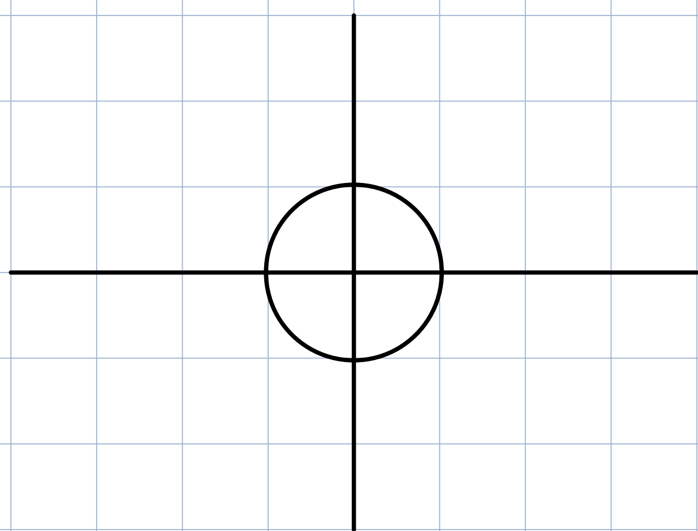
$$\Rightarrow \frac{(x-2)(x-1)}{x-3} = 1$$

↳ Solve as we did previously.

Section 5.1: The unit circle

Defn: The unit circle in the plane is

$$\{ (x,y) \mid x^2 + y^2 = 1 \}$$



↳ points on the circle

- $(1,0)$
- $(0,1)$
- $(-1,0)$
- $(\sqrt{3}/3, \sqrt{6}/3)$

$$\hookrightarrow (\sqrt{3}/3)^2 + (\sqrt{6}/3)^2$$
$$= 3/9 + 6/9$$
$$= 1$$

• $(-\sqrt{2}/2, \sqrt{2}/2)$

$$\rightsquigarrow (-\sqrt{2}/2)^2 + (\sqrt{2}/2)^2$$
$$= 2/4 + 2/4$$
$$= 1$$

Rmk: Circum of unit circle is 2π (cir $= 2\pi r$)

Given t = some real number, we can wrap starting at $(1,0)$ a dist t around the circle.

The point that we end at is called the terminal point associated to t.

$$4\overline{)29}^{\,7}, \quad 28, \quad 1$$

Ex:

i) $t = 0 \longrightarrow$ term pt $(1,0)$

ii) $t = \pi/2 \longrightarrow$ " " $(0,1)$

iii) $t = 29\pi/4 = 7\pi + \frac{\pi}{4} = 6\pi + \frac{5\pi}{4}$

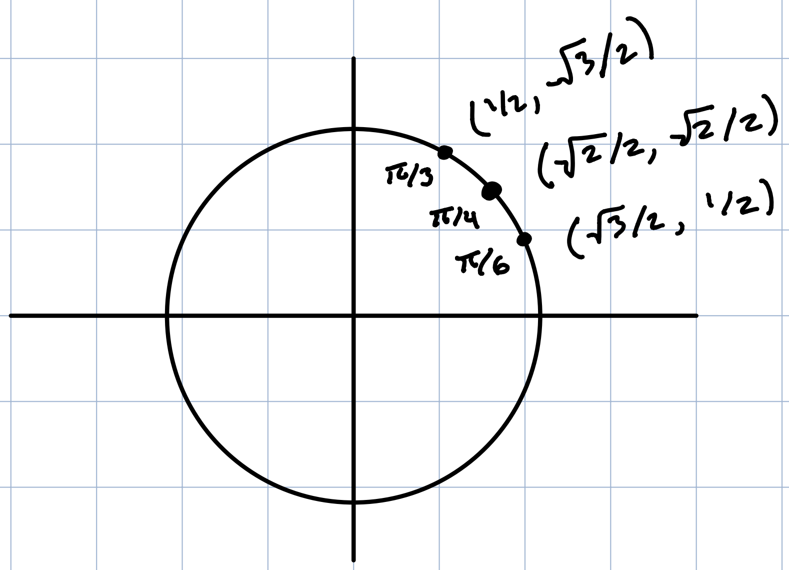
$\rightsquigarrow$ " " $(-\sqrt{2}/2, -\sqrt{2}/2)$

iv) $t = -\frac{31\pi}{4}$, what is the term. pt ass. to t.

$t = -31\pi/4 = -7\pi - \frac{3\pi}{4} = -6\pi - \frac{7\pi}{4}$

$\longrightarrow$ " " $(\sqrt{2}/2, -\sqrt{2}/2)$.

Picture:



Ex: • $t = -\pi/6$ $\to$ term pt $= (\sqrt{3}/2, -1/2)$

• $t = -3\pi/4$ $\to$ " " $= (-\sqrt{2}/2, -\sqrt{2}/2)$

• $t = 5\pi/6$ $\to$ " " $= (-\sqrt{3}/2, 1/2)$

Rmk: If $t =$ some real #, we can try deduce term pt in the following manner

1) t has same term pt as $t+2\pi$, $t-2\pi$

So find $-\pi \le t' \le \pi$ st $t' = t + 2\pi \cdot n$

where n is some integer

$\hookrightarrow$ $29\pi/2 = 14\pi + \pi/2$

$t' = \pi/2$

2) (x,y) term pt of t w/ $-\pi \le t \le 0$, then

$(x,-y)$ " " " $-t$ (so $t > 0$).

3) (x,y) " " " " w/ $\pi/2 \le t \le \pi$

$(-x,y)$ " " " $\pi - t$

$\hookrightarrow$ $3\pi/4 = t$

$\pi - 3\pi/4 = \pi/4 \to$ term $(\sqrt{2}/2, \sqrt{2}/2)$

$\Rightarrow$ term pt of $3\pi/4 = (-\sqrt{2}/2, \sqrt{2}/2)$

Ex: i) $-25\pi/6 = \frac{-24\pi}{6} - \frac{\pi}{6} = -4\pi - \frac{\pi}{6}$

$\Rightarrow$ term pt of $-25\pi/6$ = term pt of $-\pi/6$.

term pt $\pi/6$ is $(\sqrt{3}/2, 1/2)$

$\Rightarrow$ term pt $-\pi/6$ is $(\sqrt{3}/2, -1/2)$

Section 5.2:

Defn: i) $\sin(t) =$ y-coord of the terminal point of t.

ii) $\cos(t) =$ x- " " " " " " "

iii) $\csc(t) = 1/\sin(t)$

iv) $\sec(t) = 1/\cos(t)$

v) $\tan(t) = \sin(t)/\cos(t)$

vi) $\cot(t) = \cos(t)/\sin(t)$.

Ex: $\sin(0) = 0$, $\cos(0) = 1$

$\sin(\pi/6) = 1/2$, $\cos(\pi/6) = \sqrt{3}/2$

$\sin(\pi/4) = \sqrt{2}/2$, $\cos(\pi/4) = \sqrt{2}/2$

$\sin(\pi/2) = 1$, $\cos(\pi/2) = 0$

↳ use above discuss of term. pt to compute other values of sin, cos, tan, sec. ... etc.

Ex: $\cot(-\pi/3) = \dfrac{\cos(-\pi/3)}{\sin(-\pi/3)} = \dfrac{1/2}{-\sqrt{3}/2} = \dfrac{-1}{\sqrt{3}}$.

Fact: i) $\sin^2(t) + \cos^2(t) = 1$ (true b/c term pt lies on unit circle)

ii) $\tan^2(t) + 1 = \sec^2(t)$ ((i) divide by $\cos^2(t)$)

iii) $1 + \cot^2(t) = \csc^2(t)$ ((i) divide by $\sin^2(t)$)

Rmk: These formuli relate different values of trig fcns.

Ex: $\cos(t) = 3/5$ and $-\pi/2 \le t \le 0$

Question: What is $\sin(t)$?

$\hookrightarrow \sin^2(t) + 9/25 = 1$

$\Rightarrow \sin^2(t) = 16/25$

$\Rightarrow \sin(t) = \pm 4/5$

$\Rightarrow \sin(t) < 0$

$\Rightarrow \sin(t) = -4/5$

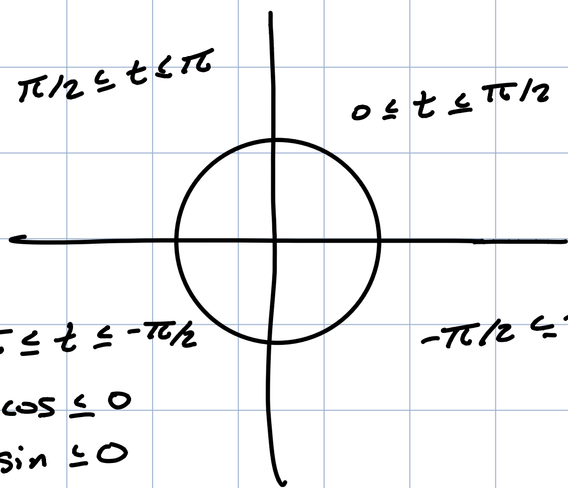
Question: " " $\cot(t)$.

$$\cot(t) = \frac{\cos(t)}{\sin(t)} = \frac{3/5}{-4/5} = \frac{-3}{4}.$$

Remark:

$\pi/2 \le t \le \pi$ — $\cos \le 0$, $\sin \ge 0$

$0 \le t \le \pi/2 \Rightarrow \sin \ge 0$, $\cos \ge 0$



$-\pi \le t \le -\pi/2$ — $\cos \le 0$, $\sin \le 0$

$-\pi/2 \le t \le 0$ — $\sin \le 0$, $\cos \ge 0$

Question: for $-\pi \le t \le -\pi/2$, what is the sign of $\sec(t)$?

$\hookrightarrow \sec(t) = 1/\cos(t)$

$\Rightarrow \cos(t) \le 0 \Rightarrow \sec(t) \le 0.$

- f is even if $f(x) = f(-x)$
- f is odd if $f(x) = f(-x)$

Remark:

Fcn	Dom	Range	even/odd
sin	$\mathbb{R}$	$[-1, 1]$	odd
cos	$\mathbb{R}$	$[-1, 1]$	even.
tan			
csc			
sec			
cot			

$-\sin(t) = \sin(-t)$

$\cos(t) = \cos(-t)$