# Lecture # 11

Warm-up: Consider $f(x) = 3x^2 + 5x - 6$. Do the following:

i) Write $f$ is standard form

ii) Write vertex of $f$

iii) " y-intercept " "

iv) " x-intercept " "

v) Does $f$ achieve a min/max value?
What is the value of $f$ and where does $f$ achieve it?

vi) What is the dom/range of $f$?

i) $f(x) = 3x^2 + 5x - 6$

$= 3\left(x^2 + \frac{5}{3}x\right) - 6$

$= 3\left(x^2 + \frac{5}{3}x + \frac{25}{36} - \frac{25}{36}\right) - 6$

$= 3\left((x + 5/6)^2 - 25/36\right) - 6$

$= 3(x + 5/6)^2 - \frac{25}{12} - \frac{72}{12}$

$= \boxed{3(x + 5/6)^2 - 97/12}$

$(x+b)^2 = x^2 + \frac{5}{3}x + a$

$\Rightarrow b^2 = a, \quad 2b = \frac{5}{3}$

$\Rightarrow b = \frac{5}{6}, \quad a = 25/36$

ii) vertex $= (-5/6, -97/12)$

iii) $y$-int $= (0, -6)$ $\quad ((0, f(0)) = y\text{-int})$

iv) $x$-int $= x$ w/ $f(x) = 0$

$\Leftrightarrow 0 = 3(x + 5/6)^2 - 97/12$

$$x = \pm\sqrt{97/36} - 5/6$$

v) $f$ achieves minimum.

$f$ achiews it when $x = -5/6$

$f(-5/6) = -97/12$

vi) $\text{dom}(f) = \mathbb{R}$

range $= [-97/12, \infty)$

## Section 3.2: Poly fcns and their graphs

Defn: A polynomial fcn $f$ is a fcn of the form

$$f(x) = a_n \cdot x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

↪ $f(x) =$ poly expression.

↪ $a_i =$ coefficients

↪ $a_0 =$ constant coeff

↪ $a_n =$ leading coeff $(a_n \neq 0)$

↪ $a_n \cdot x^n =$ leading term.

↪ $\deg(f) = n$

$$0 \cdot x^{76} + 0 \cdot x^{75} + \dots$$

Ex: $f(x) = 5x^{77} - 8x^{56} + 7x^2 - 8x + 42.$

↪ $5, -8, 7, -8, 42, 0$ are all coeffs

↪ leading coef $= 5$

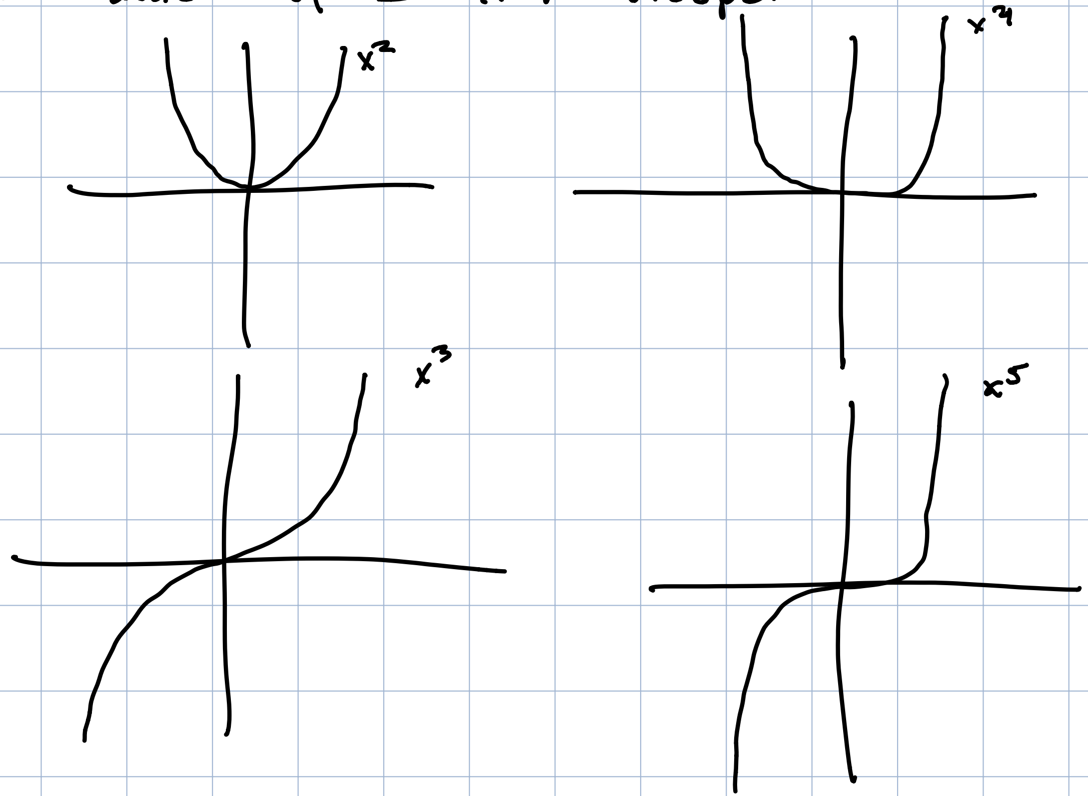
↪ constant coeff $= 42$

↪ leading term $= 5x^{77}$.

↪ $\deg(f) = 77$

Ex: $f(x) = x^n$, looks like graph of $x^2$ (n even) and $x^3$ (x odd $x > 1$), but near zero it's flatter and outside of 1 it is steeper

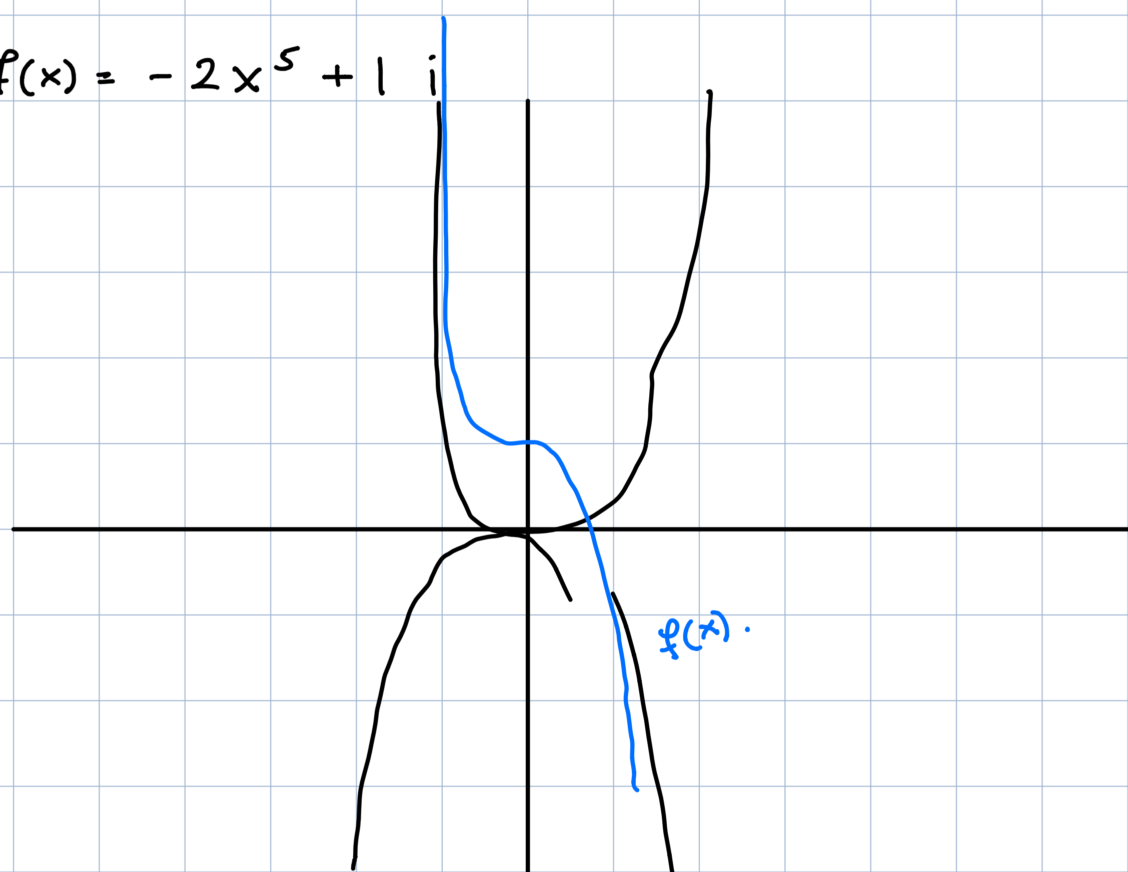
↳




↳ $x^{even}$ is even fcn (sym wrt y-axis)

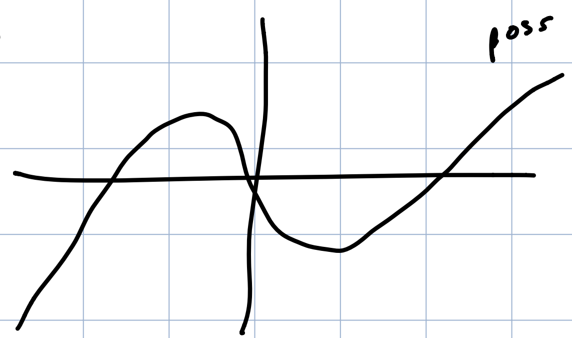
$x^{odd}$ is odd fcn (sym wrt origin)

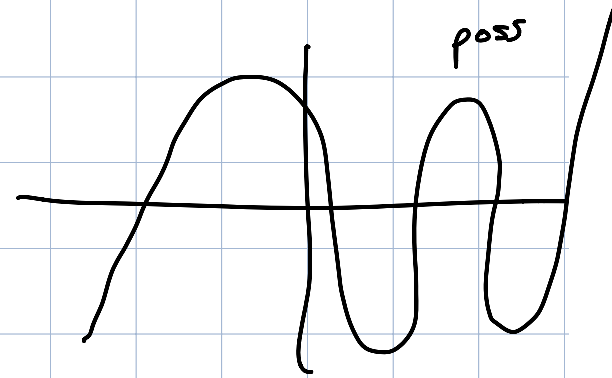
Ex: $f(x) = -2x^5 + 1$ i

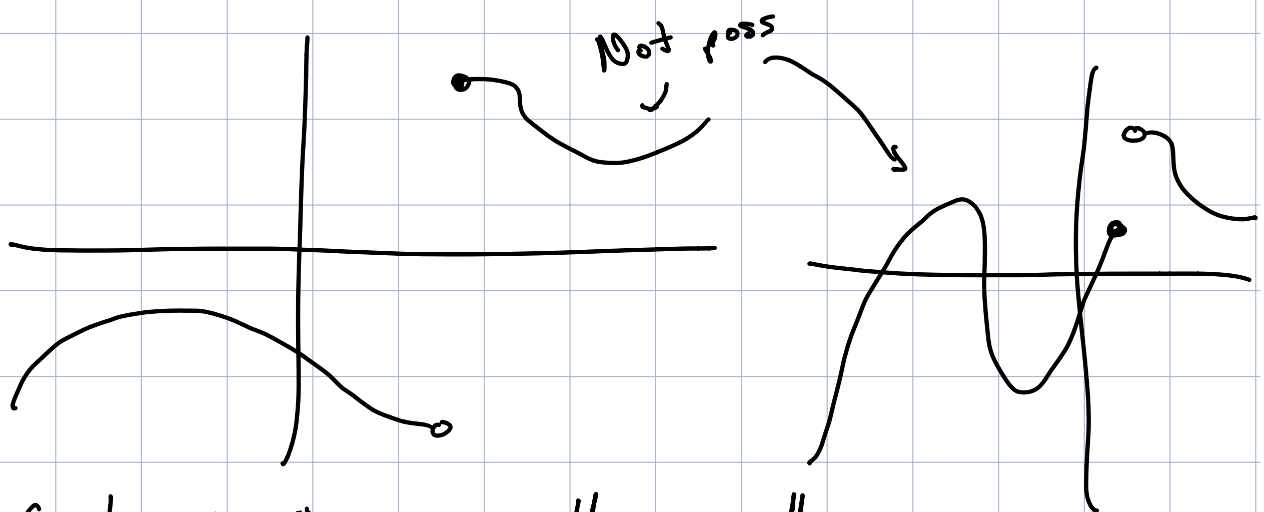


Rmk: • Graph of poly is continuous, i.e., there are no jumps. You can draw it in one stroke (ie you don't have to pick up pen).

↳

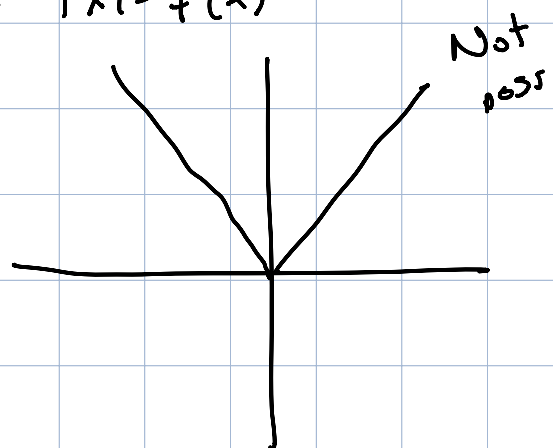


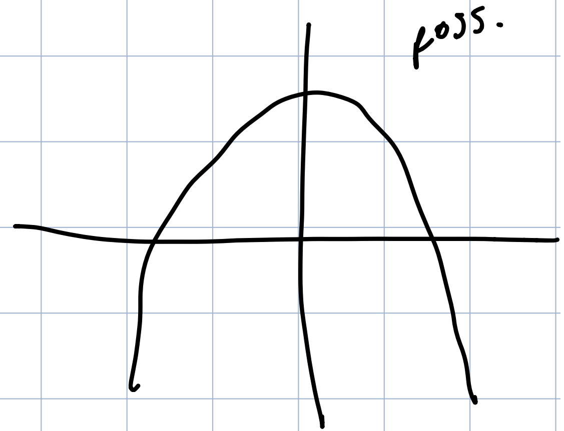



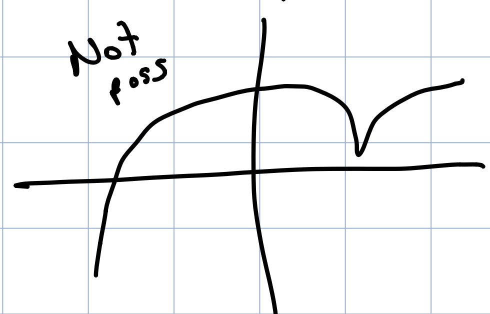

• Graphs " " is smooth, ie, there are no cusps or sharp points

↳ $|x| = f(x)$







Rmk: End behavior of poly.

↳ What happens to $f(x)$ as $x \to \pm\infty$

↳ Leading term will eventually dominate.

↳ $f(x) = x^{57} - 2x^{56} - 2x + 1$

IE end behavior of leading term is end beh. of the polynomial.

↳

| | sign leading coeff | deg $(f)$ | $x \to -\infty$ | $x \to +\infty$ |
|---|---|---|---|---|
| $x^2$ | $+$ | even | $+\infty$ | $+\infty$ |
| $-x^2$ | $-$ | even | $-\infty$ | $-\infty$ |
| $x^3$ | $+$ | odd | $-\infty$ | $+\infty$ |
| $-x^3$ | $-$ | odd | $+\infty$ | $-\infty$ |

Ex: End behavior of $f(x) = -5x^{77} + 6x^{76} + 5x^{75}$

as $x \to -\infty$, $f(x)$ goes to $+\infty$

" " " $+\infty$, " " " $-\infty$

Defn: $c$ = zero of $f$ if $f(c) = 0$, ie, graph of $f$ meets x-axis at $(c, 0)$.

Fact: The following are equivalent

i) $c$ = zero of $f$

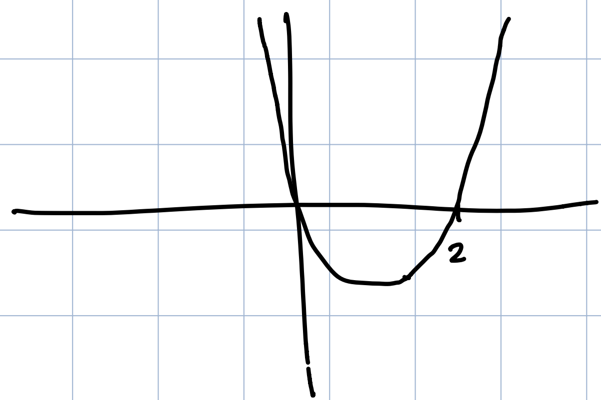
ii) $x = c$ solves $f(x) = 0$

iii) $x - c$ is a factor of $f$ $\left( f(x) = (x-c) \cdot \underset{g(x) = \text{poly}}{g(x)} \right)$

iv) graph of $f$ meets x-axis at $(c, 0)$.

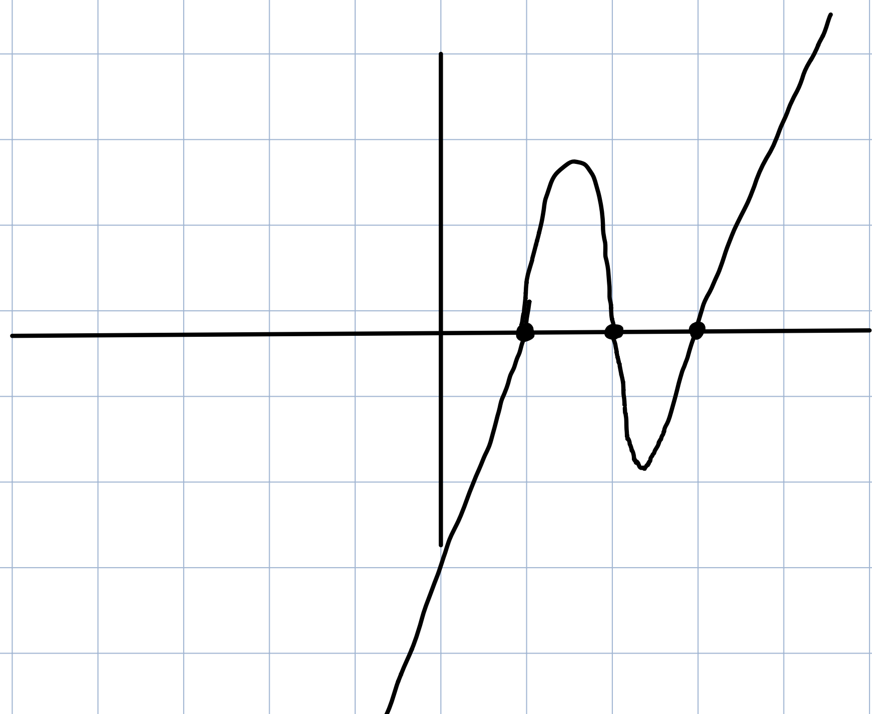
Ex: $f(x) = x^2 - 2x = x(x-2)$

$\Rightarrow$ crosses x-axis at $0, 2$



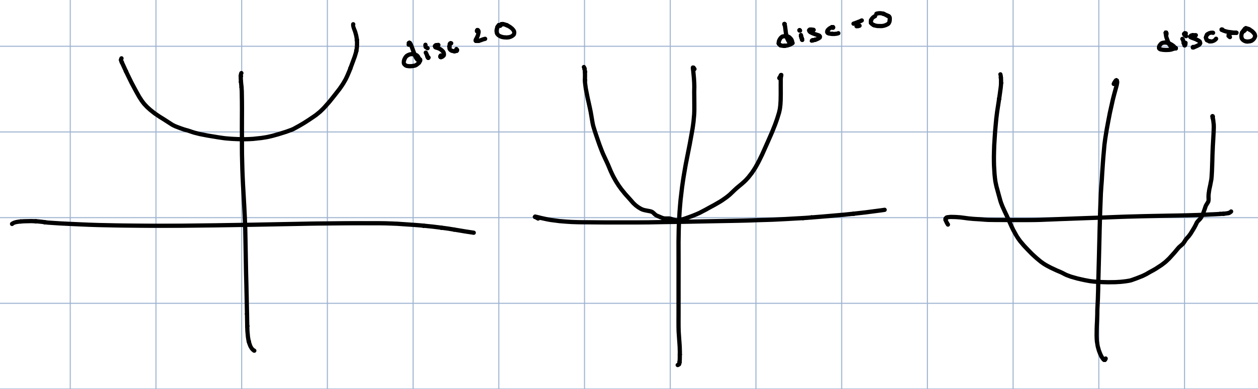

$f(x) = (x-1)(x-2)(x-3)$ $\rightsquigarrow$ mult out $3^{rd}$ deg.

$\hookrightarrow$ leading term is $x^3$

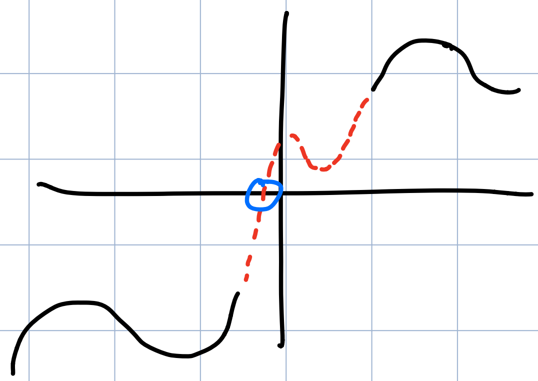


Fact: $\deg(f) = n \Rightarrow$ at most $n$ zeros.

Ex:



Fact: If $f(a)$ has opposite sign of $f(b)$, then a zero occurs between a and b



↳ between zeros the sign remains the same.

RmK: 1) Find the zeros

2) Test for sign of regions between zeros

3) Det. end behavior

Ex: $f(x) = x^3 - 2x^2 - 3x$

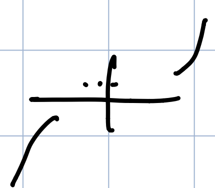
Graph $f$.

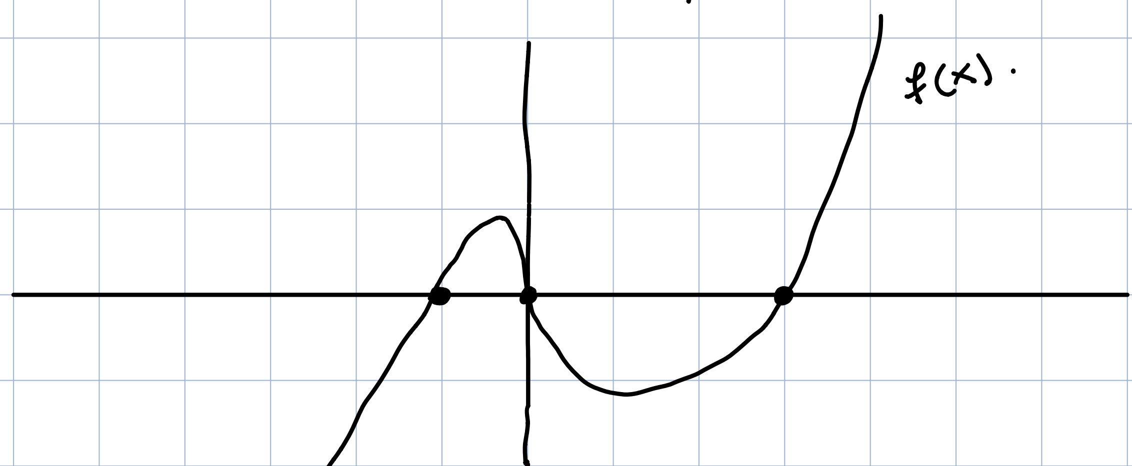
0) Factor $f(x)$

$$f(x) = x\,(x^2 - 2x - 3) = x\,(x+1)\,(x-3)$$

1) zeros $= 0, -1, 3$

3) $x^3$ = leading coeff $\Rightarrow$





Rmk: $\boxed{\text{Desmos}}$ = graphing calculator.

Defn: If $f$ has a factor $(x-c)^m$, then $f$ has a zero of order $m$ at $c$

↳ this order influences shape of graph.

↳ $f(x) = g(x)\cdot(x-c)^m$, close to $x=c$ $g(x) \neq 0$

So roughly around $x=c$, $f(x) \approx (\text{con})\cdot(x-c)^m$

⇒ graph of $f$ about $c=x$ roughly looks like the graph of $(x-c)^m$.

Ex: $f(x) = x^4(x+1)^3$

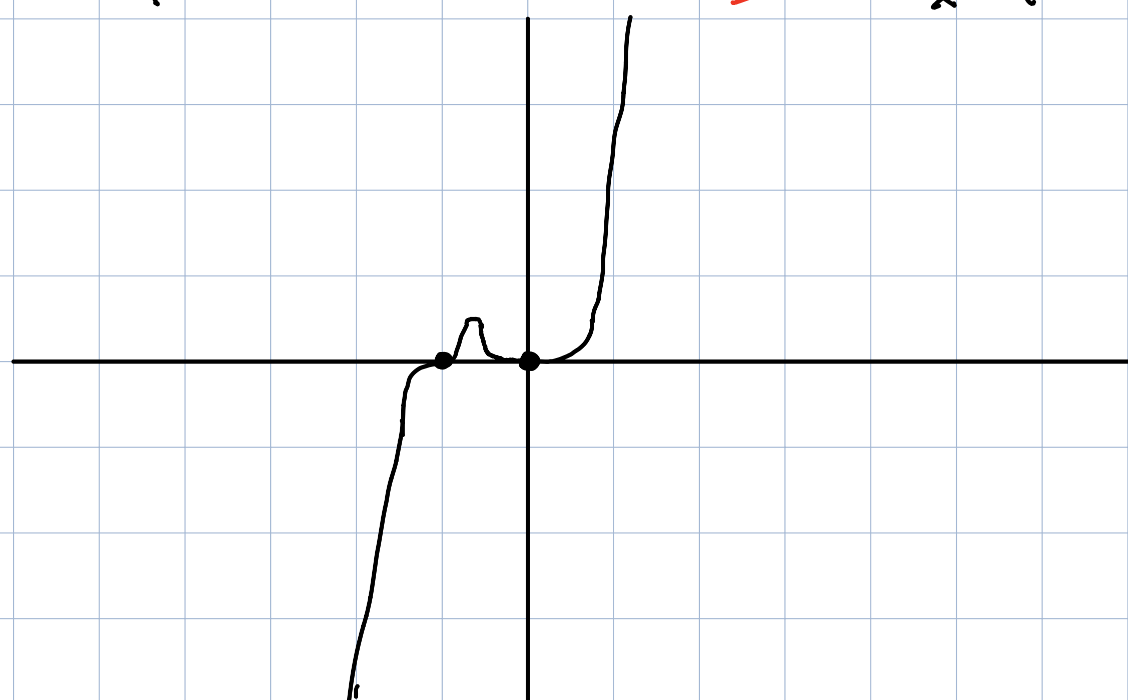
↳ $f(0) = (0)^4(0+1)^3 = 0\cdot 1 = 0.$

$f(-1) = (-1)^4(-1+1)^3 = 0.$

⇒ zeros = $0, -1$

⇒ $f$ has a zero of order 4 at $x=0$

$f$ " " " " " 3 " $x=-1$



Fact: $\deg(f) = n \Rightarrow$ graph has at most $n-1$ extrema

$\hookrightarrow$ $n-1$ bends

$\hookrightarrow$ $\deg(f) = 3$