Please read and follow the instructions below.

- Write and organize your solutions as clearly as possible. Marks may be deducted at the discretion of the grader for illegibility or poor writing.

- Working with fellow classmates and seeking help from others, if necessary, is highly encouraged, but collaboration and help should be acknowledged appropriately at the top of your solutions. The copying of answers from other students or outside sources is considered cheating.

- Students should refrain from using language like: "It is clear...", "obviously", "It is easy to show", "Clearly", etc. in their solutions.

- Given a function $f: \mathbb{R} \to \mathbb{R}$ that you know to be continuous from real analysis, you may assume without proof that $f$ is continuous here. For example, you may assume that polynomials, exp, log, square roots, etc. are continuous functions.
Question 1. (i) Show that if \( f: X \to Y \) is a homotopy equivalence and \( g: Y \to Z \) is a homotopy equivalence, then \( g \circ f \) is a homotopy equivalence.

(ii) Show that if \( f: X \to Y \) is homotopic to \( g: X \to Y \) rel \( A \subset X \) and \( g \) is homotopic to \( h: X \to Y \) rel \( A \subset X \), then \( f \) is homotopic to \( h \) rel \( A \subset X \).

Question 2. Construct an explicit deformation retraction of \( \mathbb{R}^n \setminus \{0\} \) onto \( S^{n-1} \).

Question 3. Give an example of a space \( X \) and a subspace \( A \subset X \) such that \( X \) retracts onto \( A \) but \( X \) does not deformation retract onto \( A \). (Hint: Existence of a deformation retraction implies that \( X \simeq A \).)

Question 4. Suppose that \( r: X \to X \) is a retract onto a subspace \( A \subset X \). Show that if \( X \) is contractible, then \( A \) is contractible.

Question 5. Suppose that \( f: X \to Y \) is a homotopy equivalence. Show that \( f \) gives a bijection between the path-components of \( X \) and the path-components of \( Y \).

Question 6. Show that if a CW-complex \( X \) is the union of two contractible subcomplexes whose intersection is also contractible subcomplex, then \( X \) is contractible.

Let \( X_\alpha \) be a collection of spaces indexed by \( \alpha \in \mathcal{A} \) and fix points \( x_\alpha \in X_\alpha \). The wedge sum of the \( X_\alpha \) is the space

\[
\bigvee_\alpha X_\alpha = \bigcup_\alpha X_\alpha/\sim
\]

where \( x_\alpha \sim x_\beta \) for all \( \alpha, \beta \in \mathcal{A} \). This has the effect of gluing together the spaces \( X_\alpha \) along the points \( x_\alpha \).

Question 7. Let \( X \) and \( Y \) be CW-complexes. Let \( x \in X \) and \( y \in Y \) be 0-cells. Show that \( X \vee Y \) admits the structure of a CW-complex where the wedge is obtained by \( x \sim y \).

Question 8. Show that the space \( X \) obtained from \( S^2 \) by attaching \( n \) 2-cells along \( n \) disjoint circles in \( S^2 \) is homotopy equivalent to a wedge sum of \( n+1 \) copies of \( S^2 \), that is, show that

\[
X \simeq \bigvee_{i=1}^{n+1} S^2.
\]