Please read and follow the instructions below.

- Write and organize your solutions as clearly as possible. Marks may be deducted at the discretion of the grader for illegibility or poor writing.

- Working with fellow classmates and seeking help from others, if necessary, is highly encouraged, but collaboration and help should be acknowledged appropriately at the top of your solutions. The copying of answers from other students or outside sources is considered cheating.

- Students should refrain from using language like: "It is clear...", "obviously", "It is easy to show", "Clearly", etc. in their solutions.

- Given a function $f: \mathbb{R} \to \mathbb{R}$ that you know to be continuous from real analysis, you may assume without proof that $f$ is continuous here. For example, you may assume that polynomials, exp, log, square roots, etc. are continuous functions.
Question 1. Let $X = \mathbb{R} \times \{0,1\}$ with the equivalence relation $\sim$ given by $(x,i) \sim (x,i)$ for all $(x,i)$ and $(x,0) \sim (x,1)$ if $x > 0$. Let $Y = X/\sim$ denote the quotient.

(i) Show that $Y$ is not Hausdorff.
(ii) Show that for each $y \in Y$, there exists an open subset $y \in U \subset Y$ and a homeomorphism $\phi_y : \mathbb{R} \rightarrow U$.
(iii) Show that $Y$ is second-countable.

Question 2. Let $X$ be a paracompact space. Let $A \subset X$ be a closed subspace. Show that $A$ is paracompact.

Question 3. Let $X$ be a Hausdorff space. Suppose that there exist compact subsets $K_n \subset X$ such that $K_n \subset \text{int}(K_{n+1})$ and $X = \bigcup_n K_n$. Show that $X$ is normal.

Question 4. Let $X$ be a manifold. Show that there exists a continuous function $f : X \rightarrow \mathbb{R}$ such that for every compact subset $K \subset \mathbb{R}$, $f^{-1}(K)$ is compact in $X$. (Hint: See Lecture 10 Notes bottom of page 3 and use Urysohn’s lemma.)

Question 5. Let $X$ and $Y$ be homeomorphic spaces. Show that $X$ has covering dimension $n$ if and only if $Y$ has covering dimension $n$.

Question 6. Let $X$ be a space with covering dimension less than $n$. Let $A \subset X$ be a closed subspace. Show that $A$ has covering dimension less than $n$.

Question 7. Let $X$ be a connected $T_1$ space such that $|X| > 1$. Show that $X$ has covering dimension at least 1.

Question 8. Show that $\mathbb{R}$ cannot be written as a countable union of closed subsets each having empty interior. (Hint: Baire’s theorem.)