Please read and follow the instructions below.

- Write and organize your solutions as clearly as possible. Marks may be deducted at the discretion of the grader for illegibility or poor writing.

- Working with fellow classmates and seeking help from others, if necessary, is highly encouraged, but collaboration and help should be acknowledged appropriately at the top of your solutions. The copying of answers from other students or outside sources is considered cheating.

- Students should refrain from using language like: "It is clear...", "obviously", "It is easy to show", "Clearly", etc. in their solutions.

- Given a function $f: \mathbb{R} \to \mathbb{R}$ that you know to be continuous from real analysis, you may assume without proof that $f$ is continuous here. For example, you may assume that polynomials, exp, log, square roots, etc. are continuous functions.
Question 1. Given an open cover of $S^n$ by two open subsets that are both homeomorphic to $\mathbb{R}^n$.

Question 2. Let $X$ be a connected manifold. Show that $X$ is path-connected.

Question 3. Let $X$ be a paracompact space and let $Y$ be a compact Hausdorff space. Show that $X \times Y$ is paracompact.

Question 4. If $X$ is paracompact and $f: X \rightarrow Y$ is a continuous map. Does it necessarily follow that $f(X)$ is paracompact? (c.f. If $X$ is compact and $f: X \rightarrow Y$ is continuous, then $f(X)$ is compact.)

Question 5. Is every paracompact subspace of a Hausdorff space necessarily closed? (c.f. Every compact subspace of a Hausdorff space is closed.)

Question 6. Suppose that $X$ is locally $n$-Euclidean, that is, for each $x \in X$ there is an open neighborhood $x \in U$ such that $U$ is homeomorphic to $\mathbb{R}^n$.

(i) Show that if $X$ is compact and Hausdorff, then $X$ is a manifold.

(ii) Show that the converse statement does not hold.

Question 7. Let $M_{n,n}(\mathbb{R})$ denote the set of $n$-by-$n$ matrices with real entries. Notice that $M_{n,n}(\mathbb{R}) = \mathbb{R}^{n^2}$ and equip it with the standard topology. Let $GL(n) \subset M_{n,n}(\mathbb{R})$ denote the subset of invertible matrices. Is $GL(n)$ a manifold? Either prove it or disprove it.