Reading and practice problems: Read chapters 1 and 2 from the textbook. Do exercises 1, 3, 10, 12 and 13 from chapter 1 and exercises 2, 4, 6, 8 and 10 from chapter 2. Do not hand in these exercises.

Exercises to be handed in.

Write your solutions comprehensibly.

1) Define a sequence \( \{a_n\}_{n \geq 0} \) by \( a_n = 6a_{n-1} - 9a_{n-2} \) and \( a_0 = 0, a_1 = 1 \). Prove that \( a_n = n \cdot 3^{n-1} \) is a solution to the recursion.

2) Define a sequence \( \{a_n\}_{n \geq 0} \) by \( a_{n+1} = \sqrt{\frac{12}{a_n} + 7} \) and \( a_0 = 3 \). Prove that \( a_n \leq 4 \) for every \( n \). (You may find it helpful to graph the function \( n \rightarrow a_n \) for some values of \( n \).)

3) The Fibonacci numbers \( F_n \) are defined by \( F_n = F_{n-1} + F_{n-2} \) and \( F_0 = 0, F_1 = 1 \). Prove using induction that for every \( 1 \leq m \leq n \), \( F_n = F_m F_{n+m+1} + F_{m-1} F_{n-m} \).

4) Five daffodils bloom inside a square yard of \( 1 \text{ m} \) in side length. Prove that two of the flowers are within distance \( 1/\sqrt{2} \text{ m} \).

5) A carpenter made at least 1 table per day for a period of 30 days with an average of 1.5 tables per day. Prove that there is a period of consecutive days where the carpenter built exactly 14 tables in total.

6) The Fibonacci numbers \( F_n \) are defined by \( F_n = F_{n-1} + F_{n-2} \) and \( F_0 = 0, F_1 = 1 \). Prove that there is an \( n \geq 1 \) such that 100 divides \( F_n \).