FINAL EXAM GUIDE

18.781 SPRING 2023

Topics. <u>All</u> topics that appeared on the study guides for Tests 1–3.

Also, in Stillwell, *Elements of Number Theory*:

(1) 10.1. Rings.
(2) 10.2. Fields.
(3) 10.3. Algebraic numbers and algebraic integers.
(4) 11.1–11.2. Ideals and their relation to divisibility.
(5) 11.3. Euclidean and principal ideal domains. Hence integral domains, even though Stillwell does not discuss them separately.
(6) 11.4–11.5. Examples of non-principal ideals.

The final exam will be out of 250 points. Of these, at most 25 points will be assigned to problems about rings and/or ideals.

Test Solutions.

(1) Test 1 Solutions
(2) Test 2 Solutions
(3) Test 3 Solutions

Ring theory.

Know how to...

(1) Check whether a ring is a field.
(2) Decide whether a (reasonably simple) complex number is algebraic, and if so, whether it is an algebraic integer.
(3) Check whether a subset of a ring is an ideal.
(4) For squarefree $d < 0$, plot the elements of an ideal of $\mathbf{Z}[\sqrt{d}]$ in the complex plane.
(5) Give examples of non-principal ideals in $\mathbf{Z}[\sqrt{-3}]$ and $\mathbf{Z}[\sqrt{-5}]$.
(6) Check whether a ring is an integral domain.
(7) State the definitions of Euclidean domains and principal ideal domains.
(8) Give examples of Euclidean domains and principal ideal domains.

Exercises. In Artin, *Algebra*, 2nd Edition (linked in a Canvas announcement):

(1) Chapter 11, Section 1: 1.1, 1.2, 1.6(a).
(2) Chapter 11, Section 3: 3.2, 3.3(b), 3.3(c), 3.12, 3.13.
(3) Chapter 12, Section 1: 1.4.
(4) Chapter 12, Section 2: 2.8.
(5) Chapter 13, Section 1: 1.1, 1.2.
(6) Chapter 13, Section 2: 2.1.

You can email me if you have questions about any of the problems above, or any of the problems that appeared as homework.