Problem 1. Show that the figure-8 knot is not tricolor.

Problem 2. Read about the diagrammatic definition of the linking number. Show that the link below, appropriately oriented, has linking number zero:

Problem 3. Show that for all \( n \geq 2 \), there is a braid on \( n \) strands whose closure is the unknot. It is not the identity.

Problem 4. Let \( \sigma_1 \) denote the generator of \( Br_2 \).

(1) Show that the HOMFLY invariants \( P(\sigma_1^n) \) satisfy a linear recurrence in \( n \).

(2) Deduce that the \((2, n)\)-torus links are pairwise non-isotopic.

Problem 5. Show that in \( Br_4 \), the elements

\[
(\sigma_2 \sigma_1 \sigma_3 \sigma_2)^3 \sigma_1^7 \quad \text{and} \quad (\sigma_1 \sigma_2 \sigma_3)^6 \sigma_1
\]

have the same link closure. How would you generalize this observation?

References


