18.02 HOMEWORK #9, DUE THURSDAY NOVEMBER 8TH

PART A (15 POINTS)

(11/01) Notes V2; 15.3
4C/5<u>ab</u>, 6<u>ab</u>
(11/02) 15.4 to top of page 1043.
4D/1ab<u>c</u>, <u>2</u>, <u>3</u>, <u>4</u>, 5.
(11/06) Notes V3, V4.
4E/1<u>ac</u>, <u>2</u>, 3, <u>4</u>, <u>5</u>.
4F/<u>2</u>, 3, <u>4</u>.

PART B (29 POINTS)

1. (Thursday 3 points: 1+2) Continued from question 5, Homework # 8.
(iv) Show that the curl of $\vec{F}$ is zero at any point of the plane where $\vec{F}$ is defined (not just in the right half plane $x > 0$).
(v) Is $\vec{F}$ conservative over its entire domain of definition? Is it conservative over the right half plane $x > 0$? Justify your answer.
2. (Thursday, 4 points: 2+2) (i) Calculate the curl of $\vec{F} = r^n(x\hat{\imath} + y\hat{\jmath})$ (where $r = \sqrt{x^2 + y^2}$; start by finding formulas for r_x and r_y).
(ii) Whenever possible, find a potential g such that $\vec{F} = \nabla g$ (*Hint: look for a potential of the form $g = g(r)$. Pay attention to rogue values of n*)
3. (Friday 4 points: 2+2) (i) Show that if a simple closed (positively oriented) curve C is the boundary of a region R then

$$\text{area}(R) = \oint_C x\,\mathrm{d}y = \oint_C -y\,\mathrm{d}x.$$

(ii) Find the area of the region between the x-axis and one arch of the cycloid with parametric equations

$$x = a(t - \sin t) \qquad \text{and} \qquad y = a(1 - \cos t).$$

4. (Friday 6 points: 4+2) (i) For what simple closed (positively oriented) curve C in the plane does the line integral

$$\oint_C (x^2y + y^3 - y)\,\mathrm{d}x + (3x + 2y^2x + e^y)\,\mathrm{d}y$$

have the largest possible value?
(ii) What is the maximum value?
5. (Friday 6 points: 3+3) For each statement below, say whether it is TRUE or FALSE. If it is true, explain why; if false give an example to show that it is definitely false.

(i) If $\vec{F}$ and $\vec{G}$ are conservative vector fields, then $\vec{F}+\vec{G}$ is a conservative vector field.
(ii) If M and N are differentiable functions on the region R, given by $1 < x^2 + y^2 < 4$ and $M_y(1,-1) \neq N_x(1,-1)$, then $\langle M, N \rangle$ is not a gradient vector field.
6. (Tuesday, 6 points: 2+2+2) (i) Let C be the unit circle, oriented counterclockwise, and consider the vector field $\vec{F} = xy\hat{\imath} + y^2\hat{\jmath}$. Which portions of C contribute positively to the flux of $\vec{F}$? Which portions contribute negatively?
(ii) Find the flux of $\vec{F}$ through C by direct calculation (evaluating a line integral). Explain you answer to (i).
(iii) Find the flux of $\vec{F}$ through C using Green's theorem.

Part C: 0 points