18.02 HOMEWORK #8, DUE THURSDAY NOVEMBER 1ST

Part A (16 points)

(10/25) Notes CV; 14.9, examples 1-4.
3D/$\underline{1}$,$\underline{2}$,3,$\underline{4}$.
(10/26) Notes V1; 15.2
4A/1$\underline{\text{b}}$d, 2a$\underline{\text{bc}}$, 3a$\underline{\text{b}}$c$\underline{\text{d}}$, $\underline{4}$
4B/1$\underline{\text{ab}}$cd, 2a$\underline{\text{b}}$, 3
(10/30) 15.3 to page 1033
4C/1$\underline{\text{ab}}$, 2, 3$\underline{\text{ab}}$
15.3 37

Part B (22 points)

1. (Thursday, 5 points) 3D/7.
2. (Thursday, 5 points) Find the area of the ellipse

$$(2x + 5y - 3)^2 + (3x - 7y + 8)^2 = 1.$$

3. (Friday 4 points) A *field line* for a vector field is a connected curve such that at each point of the curve, a tangent vector to the curve is parallel to the value of the vector field. Find all field lines for the plane vector field $\vec{F}(x, y) = (1 + x^2)\hat{\imath} + 4x\hat{\jmath}$.
4. (Friday 3 points) Consider the vector field $\vec{F} = (x^2y + \frac{1}{3}y^3)\hat{\imath}$ and let C be the portion of the graph of $y = f(x)$ running from $(x_1, f(x_1))$ to $(x_2, f(x_2))$ (assume that $x_1 < x_2$ and that f takes positive values). Show that the line integral

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r},$$

is equal to the polar moment of inertia of the region R lying below C and above the x-axis (with density $\delta = 1$).
5. (Tuesday, 5 points: 1+2+2) Consider the vector field

$$\vec{F}(x, y) = \frac{-y\hat{\imath} + x\hat{\jmath}}{x^2 + y^2}.$$

(i) Show that $\vec{F}$ is the gradient of the polar angle function

$$\theta(x, y) = \tan^{-1}\left(\frac{y}{x}\right).$$

defined on the right half plane $x > 0$ (*Note: this formula for θ does not make sense for* $x = 0$.)

(ii) Suppose that C is a smooth curve in the right half plane $x > 0$ joining two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$. Express

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r},$$

in terms of the polar coordinates (r_1, θ_1) and (r_2, θ_2) of A and B.
(iii) Compute directly, from the definition, the line integrals

$$\int_{C_1} \vec{F} \cdot \mathrm{d}\vec{r} \qquad \text{and} \qquad \int_{C_2} \vec{F} \cdot \mathrm{d}\vec{r},$$

where C_1 is the upper half of the unit cirle running from $(1, 0)$ to $(-1, 0)$ and C_2 is the lower half of the unit cirle running from $(1, 0)$ to $(-1, 0)$.

Part C: 0 points