18.02 HOMEWORK #6, DUE THURSDAY OCTOBER 18TH

Part A (15 points)

(10/11) Read: 13.9.
2I/1, 3.
(10/12) Read: Notes N.
2J/1, 2, 3ab, 4ab, 5ab, 6, 7.
(10/16) Read: Notes P; 14.1, focus on iterated integrals pages 942-944, 14.2, focus on evaluation of double integrals pages 950-952, 14.3 calculation of area and volume; Notes I.1.
2K/1, 2, 3, 4, 5.
3A/1, 2abc, 3ab, 4bc, 5abc, 6.

Part B (17 points)

1. (Thursday, 6 points) Find the maximum and minimum values of x^2y where x and y are positive real numbers satisfying $xy^2 = 5$.
2. (Friday, 4 points) Suppose that $g(x, y) = c$ is a constant and $w = f(x, y, z)$. Which of the following makes sense as the derivative

$$\frac{\partial w}{\partial x}?$$

(When it does make sense, compute the derivative in terms of the formal derivatives f_x, f_y, f_z, g_x and g_y. If it does not make sense, explain why it doesn't.)
(i)

$$\left(\frac{\partial w}{\partial x}\right)_x.$$

(ii)

$$\left(\frac{\partial w}{\partial x}\right)_y.$$

(iii)

$$\left(\frac{\partial w}{\partial x}\right)_z.$$

3. (Friday, 4 points: 2+2)
(i) Suppose that $t = \sin(x + y)$ and $w = x^3yt$. Find

$$\left(\frac{\partial w}{\partial t}\right)_x.$$

(ii) Consider the curve of points (x, y, z) satisfying $x^5 + yz = 3$ and $xy^2 + yz^2 + zx^2 = 7$. Use the method of total differentials to find $\frac{dx}{dy}$ at the point $(x, y, z) = (1, 1, 2)$.

4. (Tuesday, 3 points: 1+2)

Evaluate

$$\int_0^\infty \frac{e^{-x} - e^{-ax}}{x} \, \mathrm{d}x.$$

as follows:

(i) Compute

$$\int_1^a e^{-xy} \, \mathrm{d}y.$$

(ii) Use (i) to rewrite the integral we want to compute as a double integral. Evaluate the double integral by switching the order of ingegration.

Part C: 0 points

Say that a rectangle is semi-integral if the length of at least one side is a whole number. Show that if a rectangle can be subdivided into finitely many semi-integral rectangles then the original rectangle is semi-integral. (*Hint: compute*

$$\int_a^b \int_c^d \cos 2\pi x \cos 2\pi y \, \mathrm{d}x\mathrm{d}y.$$

)