18.02 HOMEWORK #5, DUE THURSDAY OCTOBER 11TH

Part A (16 points)

(10/04) Read: 13.6, pages 889-892; 13.7. (Beware of the book's annoying habit of mixing differentials $\mathrm{d}f$ with small changes in f, Δf; equations (5), (7) and (9) don't really make sense. Use (6), (8) and (10) instead).
2C/1abcd, 2, 3, 5ab.
2E/1abc, 2bc, 5, 8ab.
(10/05) Read: 13.8.
2D/1abe, 2b, 3ac, 8, 9.
2E/7.
(10/09) Holiday.

Part B (15 points)

1. (Thursday, 6 points: 1+2+2+1)
(i) Let $w = f(x, y)$ and suppose we change from rectangular to polar coordinates,

$$x = r\cos\theta$$
$$y = r\sin\theta.$$

Using the chain rule, derive the change of variables formula in matrix form:

$$A\begin{pmatrix} w_x \\ w_y \end{pmatrix} = \begin{pmatrix} w_r \\ w_\theta \end{pmatrix},$$

where the entries of the 2×2 matrix A are functions of r and θ.
(ii) Using the formulae

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1} y/x,$$

derive the changes of variables formula the other way,

$$B\begin{pmatrix} w_r \\ w_\theta \end{pmatrix} = \begin{pmatrix} w_x \\ w_y \end{pmatrix},$$

where the entries of the 2×2 matrix B are functions of x and y.
(iii) Check that $B = A^{-1}$ by computing the product AB and changing variables.
(iv) If $w_r = 2$ and $w_\theta = 10$ at the point with polar coordinates $r = 5$ and $\theta = \pi/2$, compute w_x and w_y at the same point.
2. (Friday, 4 points: 2+1+1) Let

$$f(x, y) = x^3 - xy^2 - 4x^2 + 3x + x^2 y.$$

(i) Find the maximum and minimum values of the directional derivative

$$\left.\frac{df}{ds}\right|_{\hat{u}}$$

at $(1/2, 1)$ as $\hat{u}$ varies.
(ii) In which directions $\hat{u}$ does the maximum and minimum occur?
(iii) Find the directions $\hat{u}$ for which the directional derivative is zero.
3. (Friday, 5 points: 2+3)
(i) Find the direction from $(2, -1, 1)$ for which the function $g(x, y, z) = x^2 + yz$ decreases the fastest.
(ii) Follow the line in the direction you found in part (i) to estimate, using linear approximation, the location of the point closest to $(2, -1, 1)$ at which $g = 2$. Do not use a calculator; express your answer using fractions. Next, use a calculator to evaluate g at your point. (The value should be reasonably close to 2.)

Part C: 0 points