18.02 HOMEWORK #4, DUE THURSDAY OCTOBER 4TH

Part A (8 points)

(09/27) Midterm
(09/28) Read: 13.5; Notes LS.
2F/1ab, $\underline{3}$, $\underline{4}$.
13.5/43.
2G/1$\underline{a}$b, $\underline{4}$.
(10/02) Read: 13.10; Notes SD.
2H/1a$\underline{c}$, $\underline{3}$, $\underline{4}$, $\underline{6}$.

Part B (24 points)

1. (Friday, 10 points: 2+0+3+2+3) Parts (ii-v) involve using Matlab. You have the option of using other software with similar features, or even a calculator. In this case indicate what you used and how you proceeded. You must carry out the actual calculations rather than rely on any stastistical functions built into the software.

(i) Before going to the terminal, read Notes LS and do the following. Consider the row vectors $\vec{x} = \langle x_1, x_2, \dots, x_n \rangle$, $\vec{y} = \langle y_1, y_2, \dots, y_n \rangle$ and $\vec{u} = \langle 1, 1, \dots 1 \rangle$ (n ones). Suppose that $y = ax + b$ is the equation of the line which fits the n points (x_i, y_i). Translate formula (4) of LS into a single 2×2 matrix equation

$$A\vec{z} = \vec{r} \qquad \text{where} \qquad \vec{z} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Write the entries of A and $\vec{r}$ in Matlab-ready form. Don't use summation notation instead use $\vec{x} * \vec{u}'$ (Matlab notation) for $\sum x_i$. You will be able to check that your answers are correct testing them on a correct example using Matlab in part (c).

(ii) The worldwide sale of iphones for each year from 2007 to 2011 are given below (sources:wikipedia)

Year (x_i)	0	1	2	3	4
Sales (y_i)	1.4	11.6	20.7	40.0	72.3

(To make the numerical answers easier to read we take the variable x to be the year minus 2007, so that x ranges from 0 to 4 for the given data points.)

Look at the online Matlab directions for plotting and make a scatter plot of these points marked with $*$'s. (Nothing to hand in; you will do this again in part (iv).)

(iii) Use Matlab, the data from part (ii) and the formulas you found in part (i) to find the best line $y = ax + b$ which fits the points. Compute the difference between the actual value of the data y and the predicted value $y = ax + b$. Report a, b and the worst case (largest error).

(Optional, but recommended: check that your answer for a and b agrees with the Matlab peration polyfit(x,y,1), which computes the coefficients of the best polynomial of degree 1 fitting the data x and y. If c = polyfit(x,y,1) then $\vec{c} = \langle a, b\rangle$ is the transpose of the column vector $\vec{z}$ in part (i). In this way you can confirm that you did part (i) correctly.)

(iv) When a new product is launched, in the initial period the sales tend to grow exponentially rather than linearly. Use Matlab to find the best fit of the form $\ln(y) = a_1x + b_1$. (Note: Matlab use the notation log for natural log and log 10 for log in base 10. So, in Matlab notation, you will be using $\log(y)$.) Report the values of a_1 and b_1.

If you exponentiate this equation you get

$$y = e^{b_1}e^{a_1x}.$$

Compute the difference between y and the predicted value according to this formula and report the worst case (largest error).

(v) Hand in a printout which shows on the same plot: the scatter plot of (x, y) labelled with $*$'s, the straight line fit, $(x, ax + b)$ as a dashed line and the curve $(x, e^{b_1}e^{a_1x})$, connected by an ordinary curve.

2. (Friday, 8 points: 2+2+2+2) Consider a triangle whose vertices lie on the unit circle in the plane centred at the origin. Suppose that one vertex is at $(1, 0)$ and the other two vertices have polar angles θ_1 and θ_2 in that order counterclockwise.

(i) Express the area A of the triangle in terms of θ_1 and θ_2. What is the set of possible values for θ_1 and θ_2?

(ii) Find the critical points of the function A in this region.

(iii) (Tuesday) By computing the values of A at the critical points and its behaviour on the boundary of the region (of the possible values for θ_1 and θ_2), find the maximum and minimum of A. Describe the shape of the two triangles corresponding to these two situations.

(iv) Use the second derivative test to confirm the nature of the critical points you found in (ii).

3. (Tuesday, 6 points: 4+2) Suppose that two intersecting lines are both level curves for a differentiable function $f(x, y)$.

(i) Show that the point where the two lines intersect must be a critical point.

(ii) Must this critical point be a saddle point? (If YES, explain why; if NO, give a counterexample).

Part C: 0 points

(1) To find the distance between a pair of skew lines, it was assumed that if P_1 and P_2 are the closest points then the vector $\overrightarrow{P_1P_2}$ is orthogonal to the direction of both lines. Justify this assumption.

Carry out a similar justification for the other standard distance problems (distance between a point and a plane, a point and a line, etc.).

(2) Find a function $z = f(x, y)$ and a point (x_0, y_0) such that the mixed partials, $f_{xy}(x_0, y_0)$ and $f_{yx}(x_0, y_0)$ are not equal. (*Hint: you will need a function whose second partials exist but are not continuous.*)