18.02 HOMEWORK #11, DUE THURSDAY NOVEMBER 29TH

Part A (17 points)

(11/15) Notes I4, CV.4, G; 14.7 pages 990-992.
12.8/9, 11, 15, 27, 55 (note that there is a mistake in the book; secant should be cosecant.)
5B/1abc, 2, 3, 4abc
5C/2, 3, 4.
(11/16) Notes V8, V9.
6A/1, 2, 3, 4
6B/1, 2, 3, 4, 6, 8.
(11/20) V10, 15.6
6C/1a, 2, 3, 5, 6, 7a, 8.
(11/27) V10, 15.6, pages 1054-1055.

Part B (23 points)

1. (Thursday 4 points) Write down and evaluate integrals, in both cylindrical and spherical coordinates, for the average distance from a point of the solid sphere of radius a to a fixed point O on the surface of the sphere. (*Hint: put the point O at the origin and position the sphere so the origin is the south pole of the sphere.*)
2. (Thursday 4 points) 5C/5.
3. (Friday 2 points) 6B/7.
4. (Tuesday 8 points: 1+2+3+2) Consider a tetrahedron with vertices at $P_0 = (0,0,0)$, $P_1 = (1,0,1)$, $P_2 = (1,0,-1)$ and $P_3 = (1,1,0)$.
(i) Which two faces are exchanged by the symmetry $z \longrightarrow -z$?
(ii) Find normals to each face (no need to write down unit normals) pointing outwards.
(iii) Calculate the flux of $\vec{F} = -y\hat{\imath}$ through each face.
(iv) Check the divergence theorem for this vector field and this solid, by computing each side of the formula.
5. (Tuesday 5 points: 2+1+2) Let

$$f(x,y,z) = \frac{1}{\rho} = (x^2 + y^2 + z^2)^{-1/2}.$$

(i) Calculate $\vec{F} = \nabla f$ and describe geometrically the vector field $\vec{F}$.
(ii) Evaluate the flux of $\vec{F}$ over the sphere of radius a centred at the origin.
(iii) Show that $\operatorname{div} \vec{F} = 0$. Does the answer to (ii) contradict the divergence theorem? Explain.

Part C: 0 points

Use the (3×3) Jacobian to give another way to see that

$$\Delta V \approx \rho^2 \sin\phi \Delta\theta \Delta\phi \Delta\rho.$$