FIFTH HWK, DUE THURSDAY OCTOBER 23RD

Feel free to work with others, but the final write-up should be entirely your own and based on your own understanding.

1. (15 pts) Consider the sequence

$$g_0 = 1, 1, 3, 7, 17, 41, 99, \ldots, g_n, \ldots,$$

Find a closed form expression for the nth term g_n.

2. (10 pts) (i) Find all eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

What are the corresponding eigenspaces?

(ii) Show that A is not diagonalisable.

3. (15 pts) Let

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

where $\theta \in [0, 2\pi)$ is a real number.

(i) For which values of θ is A diagonalisable over $\mathbb{R}$?

(ii) Show that A is diagonalisable over $\mathbb{C}$.

(iii) Diagonalise A over $\mathbb{C}$.

4. (10 pts) (3.1.7), page 179.

Bonus Challenge Problems:

5. (10 pts) Find all diagonalisable matrices $A \in M_{2,2}(\mathbb{F}_2)$.

6. (10 pts) (3.1.13), page 180.