FIRST HOMEWORK, DUE THURSDAY SEPTEMBER 18TH

Feel free to work with others, but the final write-up should be entirely your own and based on your own understanding.

1. (10pts) A cubic polynomial $f(t)$ in t with real coefficients has the value 12 at 2, the value 40 at 3 and the derivative $f'(t)$ has the value 5 at 1 and the value 17 at 2. Find $f(t)$. (*Warning: there are two obvious orderings of the variables, one of which gives much easier equations to solve than the other.*)

2. (10pts) Let

$$A = \begin{pmatrix} 2 & -1 & -3 \\ 4 & -3 & -1 \\ -6 & 2 & 14 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ -2 \\ 2t-1 \end{pmatrix},$$

where t is a real number. For which values of t does the system of linear equations $Av = b$ have a solution?

3. (15pts) Suppose that A is an $m \times n$ matrix, where $m < n$. For each statement below, say whether the statement is true or false. If it is false, give a counterexample and if it is true then explain why it is true.

(i) If b is any $m \times 1$ vector then the equation $Av = b$ has a solution.

(ii) The equation $Av = 0$, where 0 is the $m \times 1$ vector all of whose entries are zero, has a solution.

(iii) If b is any $m \times 1$ vector and the equation $Av = b$ has a solution then it has infinitely many solutions.