Math 3A
Spring 2016
Quiz 8
05/17/2016
Time Limit: 20 Minutes

Name (Print): ______________________________

Student ID ______________________________

This exam contains 3 pages (including this cover page) and 1 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	25	
Total:	25	

1. Let

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \tag{1}$$

with $a, b, d, c > 0$. You need to show that the two eigenvectors of $\mathbf{A}$ form a basis of $\mathbb{R}^2$. In order to make your life easier, we have split the proof in several parts.

(a) (5 points) Find the characteristic polynomial $p(\lambda)$ of $\mathbf{A}$. (**Hint:** remember that $p(\lambda) = det(\mathbf{A} - \lambda I)$.)

(b) (5 points) Find the roots of the polynomial. (**Hint:** $p(\lambda)$ is a second degree polynomial, then you can use a well known formula to compute its roots.)

(c) (5 points) Argue that if $a, b, d, c > 0$ then, the characteristic polynomial has two different roots. (**Hint:** look at the expression inside the squared root.)

(d) (5 points) Argue, using a theorem in your book, that the eigenvectors are linear independent.

(e) (5 points) Using the basis Theorem, conclude that the two eigenvectors of $\mathbf{A}$ form a basis of $\mathbb{R}^2$