Math 2B multiple choice sample questions, Spring 2017

1. (Section 4.9) The function $F(x)$ satisfies $F'(x) = 3x(x-2)$ and $F(0) = 1$. What is $F(1)$?
   a. -3
   b. -3/2
   c. -1
   d. 0
   e. 3/2

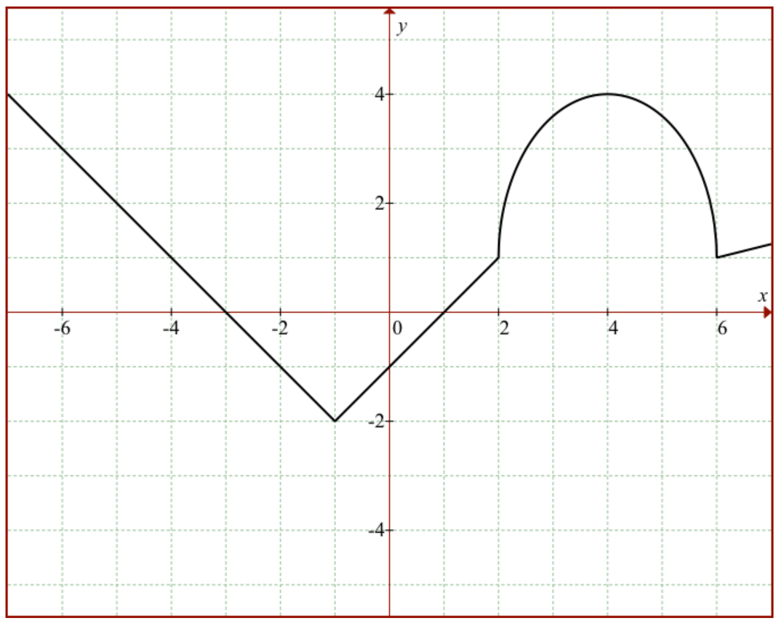

Figure 1. This shows the graph of a function $f(x)$ referred to in Questions 2 and 3.

2. (Section 4.9) Let $F(x)$ denote an antiderivative of $f(x)$, where $y = f(x)$ is shown in Figure 1. Which of the following can we deduce about $F(-5)$?
   a. We have $F(-5) > 0$, because $f(-5) > 0$.
   b. We have $F(-5) < 0$, because $f'(-5) < 0$.
   c. We have $F(-5) > 0$, because $f'(-5) > 0$.
   d. We cannot deduce any information about whether $F(-5)$ is positive or negative.

3. (Section 5.2) Figure 1 shows the graph of a function $y = f(x)$. Imagine we estimate both of the integrals $\int_{-6}^{-4} f(x)\,dx$ and $\int_{2}^{4} f(x)\,dx$ using Riemann sums with 20 rectangles and left endpoints. Which of the following is true?
   a. The estimate of $\int_{-6}^{-4} f(x)\,dx$ is an under-estimate and the estimate of $\int_{2}^{4} f(x)\,dx$ is an over-estimate.

b. The estimate of $\int_{-6}^{-4} f(x)\,dx$ is an over-estimate and the estimate of $\int_{2}^{4} f(x)\,dx$ is an under-estimate.

c. The estimates of $\int_{-6}^{-4} f(x)\,dx$ and $\int_{2}^{4} f(x)\,dx$ are both over-estimates.

d. The estimates of $\int_{-6}^{-4} f(x)\,dx$ and $\int_{2}^{4} f(x)\,dx$ are both under-estimates.

4. (Section 5.2) Define the numbers A and B as follows:

$$A = \int_0^{10} \left|x^2 - 10x + 3\right|\,dx \text{ and } B = \int_0^{10} \left|x^2 + 10x - 3\right|\,dx.$$

Which of the following statements is true?

a. $A \geq 0$ and $B \leq 0$

b. $A \leq 0$ and $B \leq 0$

c. $A \geq 0$ and $B \geq 0$

d. $A \leq 0$ and $B \leq 0$

5. (Section 5.3) Let $f(x) = \int_x^3 \sin(2t)\,dt$. Compute $f'(x)$.

a. $f'(x) = -\sin(2x)$

b. $f'(x) = \sin(6) - \sin(2x)$

c. $f'(x) = -2\cos(2x)$

d. $f'(x) = \frac{1}{2}\cos(2x)$

6. (Section 5.4) A wolf population begins with 100 wolves and increases at a rate of $n'(t)$ wolves per week. What does the quantity

$$100 + \int_0^8 n'(t)\,dt$$

represent? No explanation is necessary.

a. The average number of wolves in the population during the first 8 weeks.

b. The average rate of change of the wolf population over the first 8 weeks.

c. The total number of wolves in the wolf population after the first 8 weeks.

d. The number of wolves gained by the wolf population during the first 8 weeks.

7. (Section 5.5) Compute $\int \frac{1/2}{x+1}\,dx$.

a. $\ln(x+1) + \frac{1}{2} + C$

b. $\frac{1}{2}\ln(x) + C$

c. $\ln\sqrt{x+1} + C$

d. $\frac{-1}{2(x+1)^2} + C$

8. (Section 5.5) Compute $\int_0^1 e^{x+e^x}\,dx$.

a. $e(e^{e-1}-1)$

b. $e^{e^{e^e}}$

c. e^{e-1}

d. e^e

e. $(e-1)e^{e-1}$

9. (Section 6.1) Which of the following represents the area between the two curves $y=\sin(x)$ and $y=\cos(x)$ in the interval $0 \le x \le \frac{\pi}{2}$?

a. $\displaystyle\int_0^{\pi/2} \Big(\sin(x)-\cos(x)\Big)\,dx$

b. $\displaystyle\int_0^{\pi/2} \Big(\cos(x)-\sin(x)\Big)\,dx$

c. $\displaystyle\frac{1}{\pi/2}\int_0^{\pi/2} \Big(\sin(x)+\cos(x)\Big)\,dx$

d. $\displaystyle\int_0^{\pi/4} \Big(\cos(x)-\sin(x)\Big)\,dx + \int_{\pi/4}^{\pi/2} \Big(\sin(x)-\cos(x)\Big)\,dx$

10. (Section 6.2) The definite integral $\int_0^4 \pi y\,dy$ represents the volume of which of the following solids?

a. The region bounded by the y-axis, $y=x^2$, and $y=2$, rotated about the y-axis

b. The region bounded by the y-axis, $y=x^2$, and $y=4$, rotated about the y-axis

c. The region bounded by the x-axis, $y=\sqrt{x}$, and $x=2$, rotated about the x-axis

d. The region bounded by the x-axis, $y=\sqrt{x}$, and $x=16$, rotated about the x-axis

11. (Section 6.5) Which of the following represents the average of the function $f(x)=\cos^2(x^2)$ over the interval from $x=0$ to $x=\pi/2$?

a. $\frac{2}{\pi}\int_0^{\pi/2} f(x)\,dx$

b. $\int_0^{\pi/2} f'(x)\,dx$

c. $\displaystyle\frac{f(\pi/2)-f(0)}{\pi/2}$

d. $\sqrt{f(\pi/2)f(0)}$

12. (Section 7.1) Using integration by parts, we see that $\int x\ln x\,dx$ is equal to which of the following?

a. $\displaystyle\frac{x^2\ln x}{2} - \int \frac{x}{2}\,dx$

b. $\dfrac{x^3}{2} - \displaystyle\int \dfrac{x^2}{2}\,dx$

c. $\dfrac{x^3 \ln x}{2} - \displaystyle\int 1\,dx$

d. $\dfrac{x^2}{2} - \displaystyle\int \ln x\,dx$

13. (Section 7.3) While solving a trigonometric substitution question, we find $x = \tan\theta$, where $0 < \theta < \pi/2$. Which of the following is equal to $\cos(\theta)$?

a. $\sqrt{x^2 - 1}$

b. $\frac{1}{x} + \frac{1}{x+1}$

c. $\frac{1}{\sqrt{x^2+1}}$

d. $\frac{x^2-1}{\sqrt{2}}$

14. (Section 7.3) To compute the definite integral $\int_0^2 \sqrt{9 - x^2}\,dx$, which of the following substitutions could be used?

a. $x = 3\sin(\theta)$ and $dx = 3\cos(\theta)\,d\theta$

b. $x = 3\tan(\theta)$ and $dx = 3\sec^2(\theta)\,d\theta$

c. $x = 3\sec(\theta)$ and $dx = 3\sec(\theta)\tan(\theta)\,d\theta$

d. $x = 9 - \theta^2$ and $dx = -2\theta\,d\theta$

15. (Section 7.8) What is wrong with the computation

$$\int_{-1}^{1} \frac{1}{x}\,dx = \ln|x|\Big]_{-1}^{1} = \ln(1) - \ln(1) = 0?$$

a. The function $\ln|x|$ is not an antiderivative of $\frac{1}{x}$.

b. The function $\frac{1}{x}$ has an asymptote at $x = 0$ so we should have used an improper integral.

c. We are missing a "$+C$", so the final answer should be $0 + C = C$.

d. The value $\ln(1)$ is not defined, so we can't say $\ln(1) - \ln(1) = 0$.

16. (Section 11.4) Consider the series

$$A\colon \sum_{k=1}^{\infty} \frac{1}{2k-1} \text{ and } B\colon \sum_{k=1}^{\infty} \frac{1}{3k+1}.$$

Which of the following is the true statement?

a. Both series converge.

b. Both series diverge.

c. Series A converges and series B diverges.

d. Series A diverges and series B converges.

17. (Section 11.8) What is the interval of convergence of $\sum_{k=1}^{\infty} \frac{1}{2k} x^k$?

a. $x = 0$

b. $-2 < x \leq 2$

c. $-2 \leq x < 2$

d. $-1 < x \leq 1$

e. $-1 \leq x < 1$

18. (Section 11.8) What is the interval of convergence of $\sum_{k=0}^{\infty} \frac{k}{7^k} x^k$?

a. $x = 0$

b. $-7 < x < 7$

c. $-7 \leq x < 7$

d. $-7 < x \leq 7$

e. $-7 \leq x \leq 7$

19. (Section 11.9) Which of the following is the power series representation of $\frac{2}{2+x}$?

a. $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

b. $\sum_{n=0}^{\infty} \left(\frac{-x}{2}\right)^n$

c. $\sum_{n=0}^{\infty} 2\left(\frac{x}{2}\right)^n$

d. $\sum_{n=0}^{\infty} \frac{(-x)^n}{2}$

20. (Section 11.10) Determine the value of $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$.

a. 0

b. $-e$

c. $\frac{1}{e}$

d. $\cos(e)$