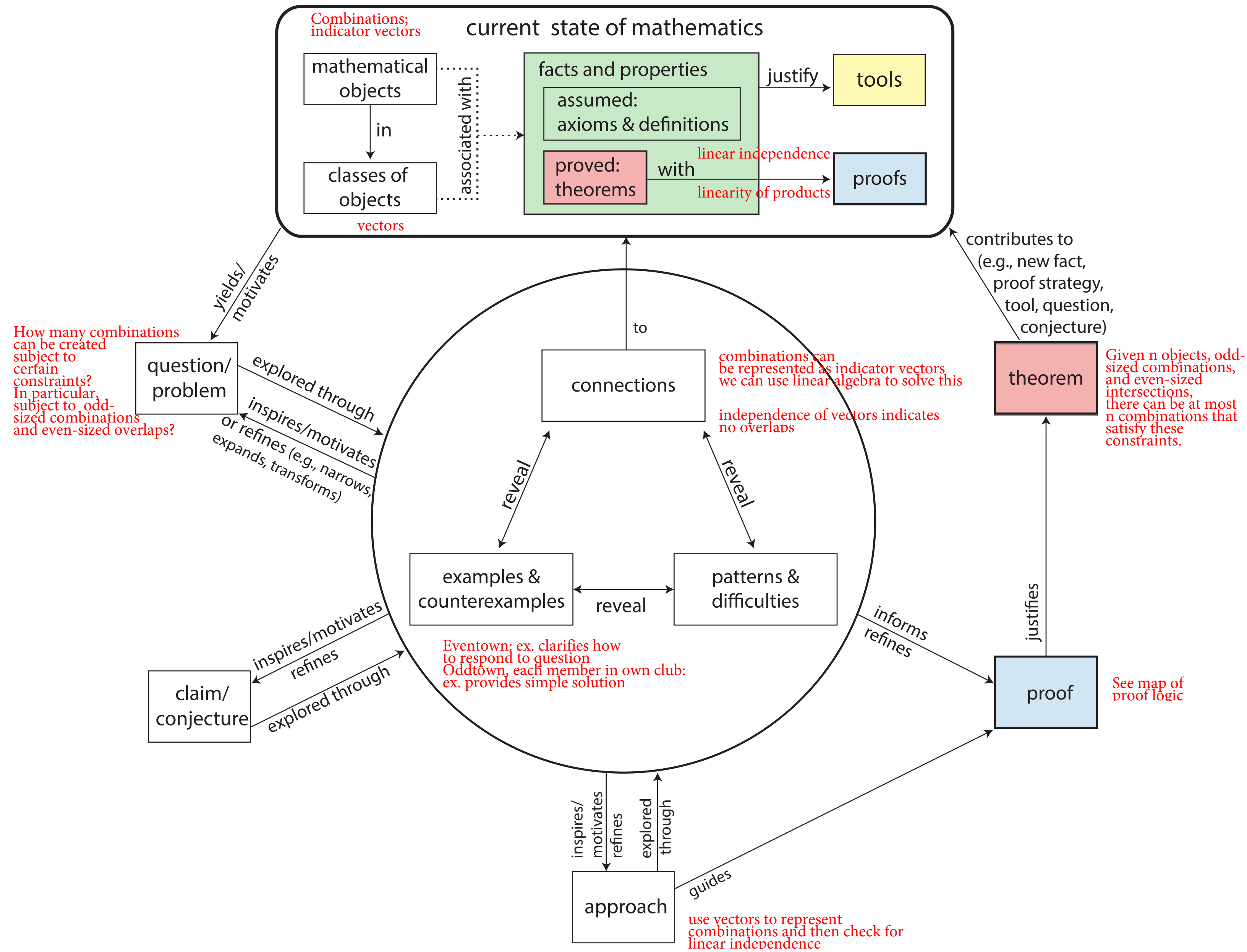
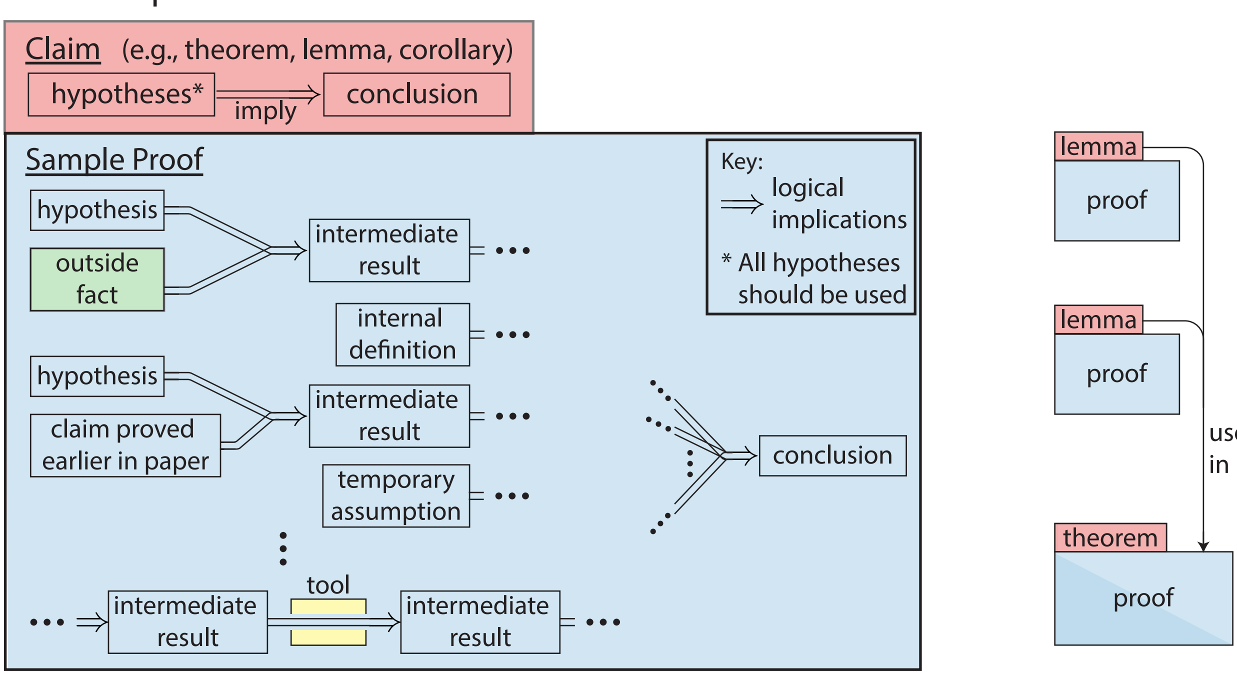
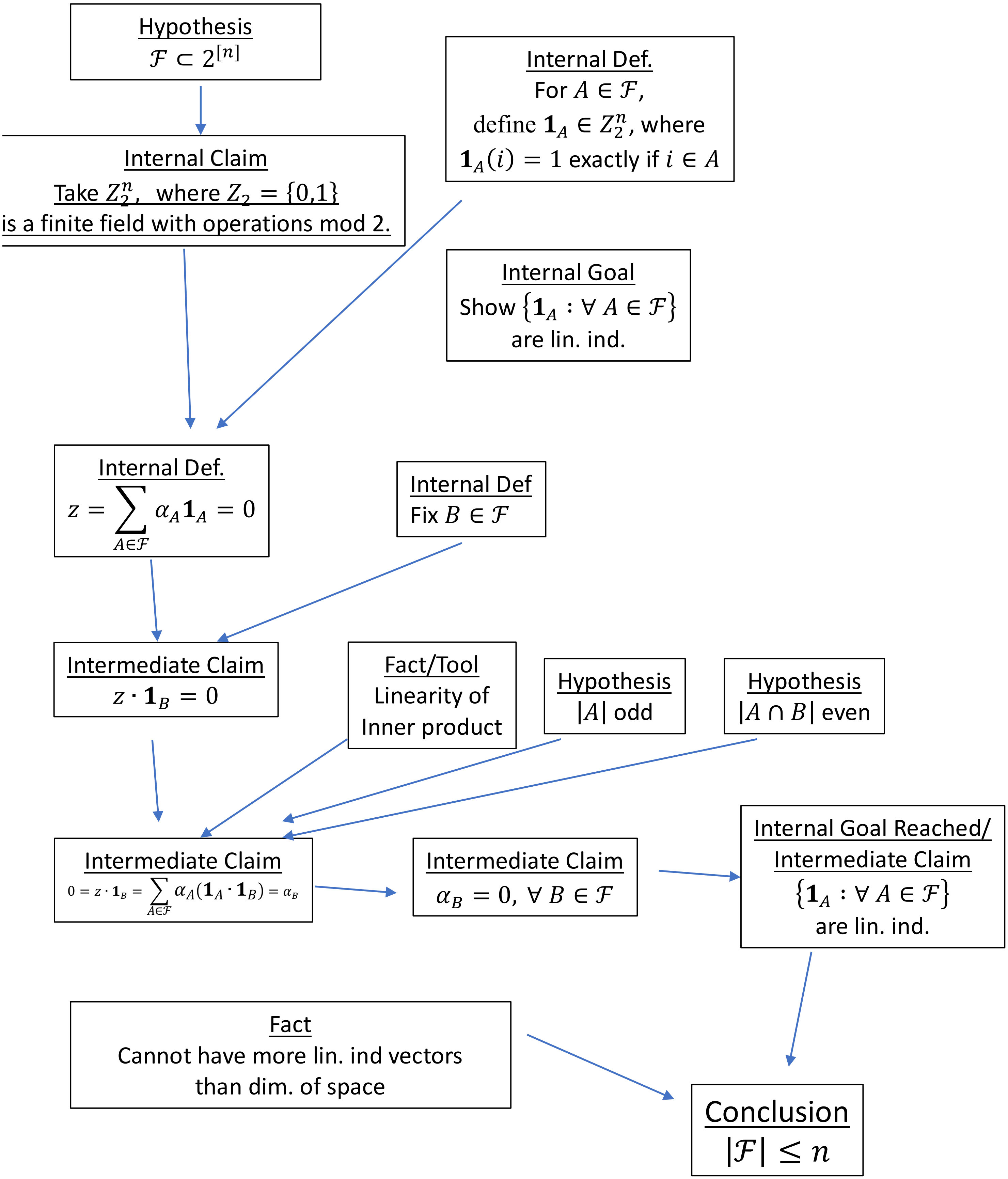
# Mathematics Reasoning Diagram




## What is a proof?



PROOF:


Hypothesis
$\mathcal{F} \subset 2^{[n]}$
Internal Claim
Take $Z_2^n$, where $Z_2 = \{0,1\}$
is a finite field with operations mod 2.
Internal Def.
For $A \in \mathcal{F}$,
define $\mathbf{1}_A \in Z_2^n$, where
$\mathbf{1}_A(i) = 1$ exactly if $i \in A$
Internal Goal
Show $\{\mathbf{1}_A : \forall A \in \mathcal{F}\}$
are lin. ind.
Internal Def.
$z = \sum_{A \in \mathcal{F}} \alpha_A \mathbf{1}_A = 0$
Internal Def
Fix $B \in \mathcal{F}$
Intermediate Claim
$z \cdot \mathbf{1}_B = 0$
Fact/Tool
Linearity of
Inner product
Hypothesis
$|A|$ odd
Hypothesis
$|A \cap B|$ even
Intermediate Claim
$0 = z \cdot \mathbf{1}_B = \sum_{A \in \mathcal{F}} \alpha_A (\mathbf{1}_A \cdot \mathbf{1}_B) = \alpha_B$
Intermediate Claim
$\alpha_B = 0, \forall B \in \mathcal{F}$
Internal Goal Reached/
Intermediate Claim
$\{\mathbf{1}_A : \forall A \in \mathcal{F}\}$
are lin. ind.
Fact
Cannot have more lin. ind vectors
than dim. of space
Conclusion
$|\mathcal{F}| \leq n$