What is a proof?

**Claim** (e.g., theorem, lemma, corollary)

- **hypotheses**
  - imply
  - **conclusion**

**Sample Proof**

- **hypothesis**
  - intermediate result
    - internal definition
      - claim proved earlier in paper
        - temporary assumption
          - tool
            - intermediate result
              - intermediate result

- **outside fact**
  - intermediate result
    - internal definition
      - claim proved earlier in paper
        - temporary assumption
          - tool
            - intermediate result
              - intermediate result

**Key:** logical implications

*All hypotheses should be used*

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**Example:**

**Theorem**

- proof
  - lemma
    - proof
  - lemma
    - proof
  - **theorem**
    - proof

---

**Connections:**

- **current state of mathematics**
  - mathematical objects
    - classes of objects
      - associated with
        - facts and properties
          - justified
            - tools
  - **proves:**
    - theorems
      - with
        - linear independence
          - linearity of products

**Examples & Counterexamples**

- intermediate result
  - intermediate result
    - intermediate result
      - intermediate result
        - conclusion

**Patterns & Difficulties**

- intermediate result
  - intermediate result
    - intermediate result
      - intermediate result
        - conclusion

**Approach**

- intermediate result
  - intermediate result
    - intermediate result
      - conclusion

**Theorem**

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  - lemma
    - proof
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PROOF:

**Hypothesis**
\[ \mathcal{F} \subset 2^{[n]} \]

**Internal Claim**
Take \( Z_2^n \), where \( Z_2 = \{0,1\} \) is a finite field with operations mod 2.

**Internal Def.**
For \( A \in \mathcal{F} \), define \( \mathbf{1}_A \in Z_2^n \), where 
\[ \mathbf{1}_A(i) = 1 \text{ exactly if } i \in A \]

**Internal Goal**
Show \( \{ \mathbf{1}_A : \forall A \in \mathcal{F} \} \) are lin. ind.

**Internal Def.**
\[ z = \sum_{A \in \mathcal{F}} \alpha_A \mathbf{1}_A = 0 \]

**Internal Def**
Fix \( B \in \mathcal{F} \)

**Intermediate Claim**
\[ z \cdot \mathbf{1}_B = 0 \]

**Fact/Tool**
Linearity of Inner product

**Hypothesis**
\[ |A| \text{ odd} \]

**Hypothesis**
\[ |A \cap B| \text{ even} \]

**Intermediate Claim**
\[ 0 = z \cdot \mathbf{1}_B = \sum_{A \in \mathcal{F}} \alpha_A (\mathbf{1}_A \cdot \mathbf{1}_B) = \alpha_B \]

**Intermediate Claim**
\[ \alpha_B = 0, \forall B \in \mathcal{F} \]

**Intermediate Claim**
\[ \mathbf{1}_A : \forall A \in \mathcal{F} \] are lin. ind.

**Fact**
Cannot have more lin. ind vectors than dim. of space

**Conclusion**
\[ |\mathcal{F}| \leq n \]