18.204 Term Paper Workshop

18.204 SPRING 2020

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Paper requirements

- For the term paper, you will write a paper based on your topic.
- The paper should be at least 10 pages, and should include an introduction, sections in the body, a conclusion, and citations.
- The final version will be due at the end of the semester. The paper will be written in various stages throughout the semester.
- The paper does not need original research.
- The paper should be your own product, with proper citations for all background, theorems, and proofs.

Paper deliverables

- **March 30:** Skeleton with outline/structure of paper, background on the topic, one proposition/lemma/theorem proof.
- **April 13:** First version of paper.
- **April 27:** Next version of paper, revised based on comments from me.
- **May 4:** Peer review two other students' papers, due in class.
- **May 11:** Final version of paper due by end of day.

March 30: Skeleton

- Submit a skeleton of a paper on Stellar
- Draft of an introduction that explains in particular the background and history, and motivation of the topic.
- Outline and plan for sections, theorems, lemmas, etc.
- Include detailed write-up of one full proof (at least half a page), with citation for where the proof is from.
- I will send out a template latex file to use.

April 13: First version

- Submit the first version of the paper on Stellar.
- This should be a full version of the paper, with an introduction, sections, conclusion, and references.

April 27: Second version

- Submit a revised version of the paper on Stellar, based on feedback from me on the first version.
- This should be a full version of the paper, with an introduction, sections, conclusion, and references.
- This will be sent out to other students for peer review.

May 4: Peer review

- Before class, you will review 2-3 other students' papers.
- Class will be a peer review workshop, where students will discuss each others' papers and give comments and feedback.

May 11: Final version

- The final version of the paper will be submitted on Stellar by end of day.
- Make sure you use the template I send out.

Citations and fair use

ADAPTED FROM SLIDES OF ZILIN JIANG

What information do mathematicians want from references?

- Which results are found in a given reference?
- Who is responsible for the results and where can these results be found?
- If not provided in the current piece, where is the best place to see a proof of these results?

*And they want to see it as efficiently as possible.

[SLIDE FROM ZILIN JIANG]

Which results are found in a given reference?

- Consider the following example:

  In 1986, Frankl and Rödl proved that the vertex set of all triangles is Ramsey, and in 1990 extended this result to the vertex set of every non-degenerate simplex in any dimension [FR90, FR86].

- What does the citation and its ordering mean?

  Are both results in [FR90]? What is in [FR86]?

- Order and locate sources so their relationship to given result(s) is(are) clear, e.g.:

  In 1986, Frankl and Rödl [FR86] proved that the vertex set of all triangles is Ramsey, and in 1990 extended in [FR90] this result to the vertex set of every non-degenerate simplex in any dimension.

[SLIDE FROM ZILIN JIANG]

Which results are found in a given reference?

- Consider the following caption as an example:

  Figure 1: [ER60] An example of three different Erdős-Rényi random graphs with different values for p.

- What content is found in [ER60]?

  The example itself? The result illustrated by the example? The figure itself?

- Write so the reader knows exactly what content is found in the referenced source, e.g.:

  Figure 1: An example of three different Erdős-Rényi random graphs with different values for p (figures taken from [ER60]).

[SLIDE FROM ZILIN JIANG]

Which results are found in a given reference?

- Consider the following example:

  Next, we consider a specific subset of planar graphs, Hamiltonian graphs, and how their relationship to triangular planar maps—and, subsequently, cubic graphs—allows for interesting properties related to the four-color problem [Whi31] [Saa72].

- What results are being cited?
  - consider a specific subset of planar graphs, Hamiltonian graphs?

  considering a specific subset isn't a result

  - how their relationship to triangular planar maps—and, subsequently, cubic graphs—allows for interesting properties

  "how..." isn't a result

  "relationship" & "interesting properties" are not specific enough to be results

- You shouldn't need to cite your own organizational strategies!

[SLIDE FROM ZILIN JIANG]

Who is responsible for the results?

- Consider the following:

  Another conjecture is that the maximal number of equiangular lines in R^r must be even [Red09].

- Who is responsible for the conjecture?

  [Red09]? Someone referenced in [Red09]? Who is [Red09], anyway?

- Name the author of the result in the sentence to avoid such confusion, e.g.:

  Redmond [Red09] made another conjecture that the maximum number of equiangular lines in R^r must be even.

[SLIDE FROM ZILIN JIANG]

Who is responsible for the results? (Where is the best place to see a proof of these results ?)

- Consider the following:

  We can show that v(r) is at most (r+1 choose 2), known as the absolute bound [LS73], [Mat10].

- To whom is this result due?

  The author? LS73? Mat10? Who is/are LS73?

- Attribute results so we know what kind(s) of results are due to whom and in which reference each can be found, e.g.:

  Lemmens and Seidel [LS73] showed that v(r) is at most (r+1 choose 2), known as the absolute bound (see [Mat10] for a short proof).

*Note that you need to decide the best place to direct readers for a proof, as it is not always the initial source of the results.

[SLIDE FROM ZILIN JIANG]

Efficiency

- Consider the following example:

  The technical details are not given here but can be found in Spencer's paper ([Spe85]).

- The reference is redundant information, as Spencer's paper is signaled more precisely in the reference.
- Write to avoid such redundancy by naming the source in the sentence, e.g.:

  The technical details are not given here but can be found in [Spe85].

[SLIDE FROM ZILIN JIANG]

Fair use

- You are not expected to come up with original theorems and proofs for the term paper.
- You can copy theorem statements, figures, if they are properly cited. Eg.

  Theorem 2. [Theorem 1 in ABC12] Given a graph G=(V,E) and...
- Occasionally you will see direct quotes in other situations in math papers (mostly in the introduction), but it is not common. In this case, clearly indicate that you are quoting, and put a citation.
- The structure, introduction, guiding words, and explanations should be your own work.
- Think about: what is the main goal, core concept, of YOUR paper? It will be different from that of your sources.

Fair use

- Cite where the proof comes from, but write the proofs yourself.

  This was originally proved by XYZ in [XYZ12]. Here we follow the proof strategy in [ABC12]....

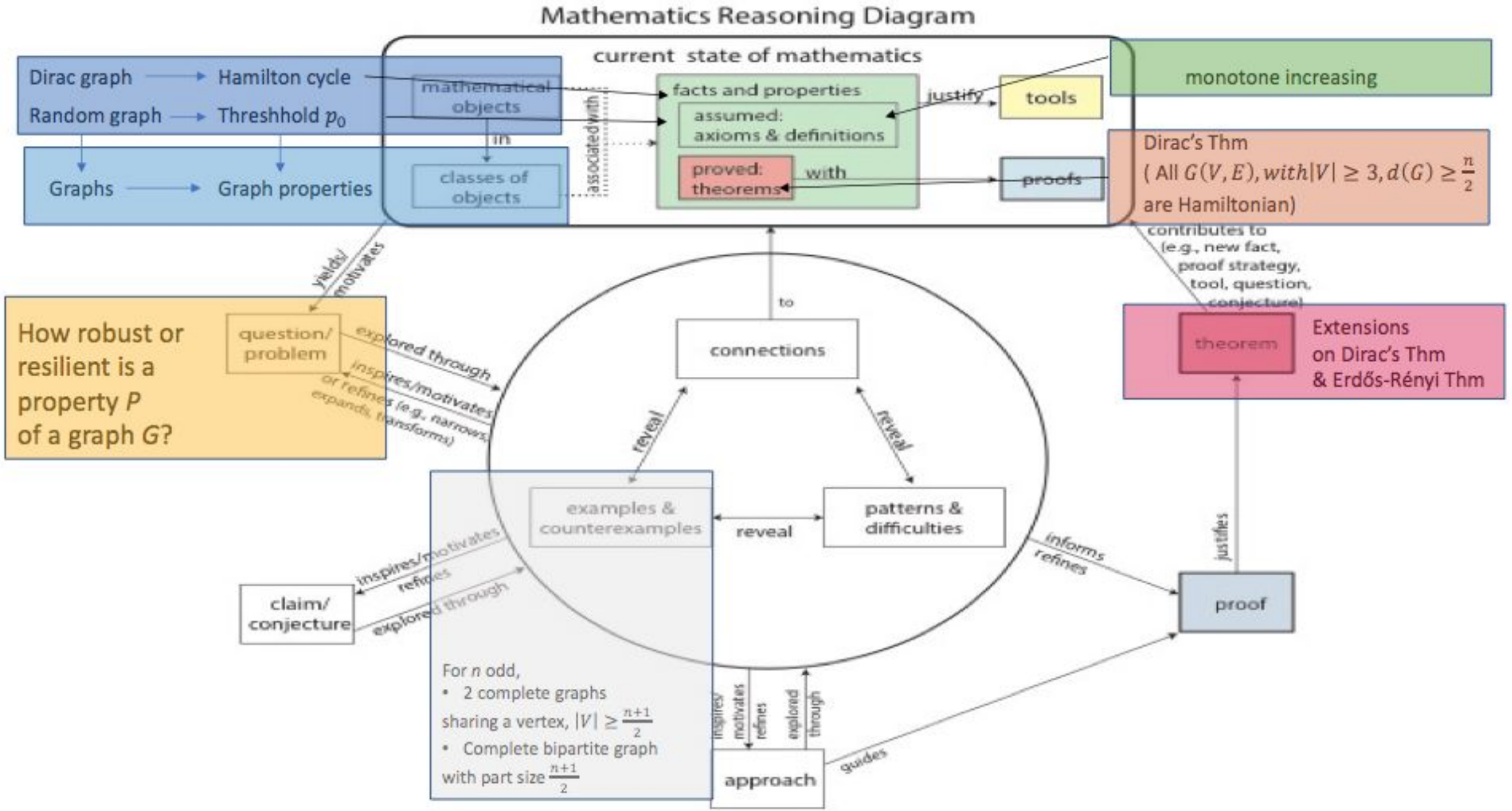
- Having essentially same sentences as a source, each sentence slightly rewritten, is NOT acceptable.
- The best strategy is to:
  - Read and understand the proof.
  - Put the original source away.
  - Write up the proof.

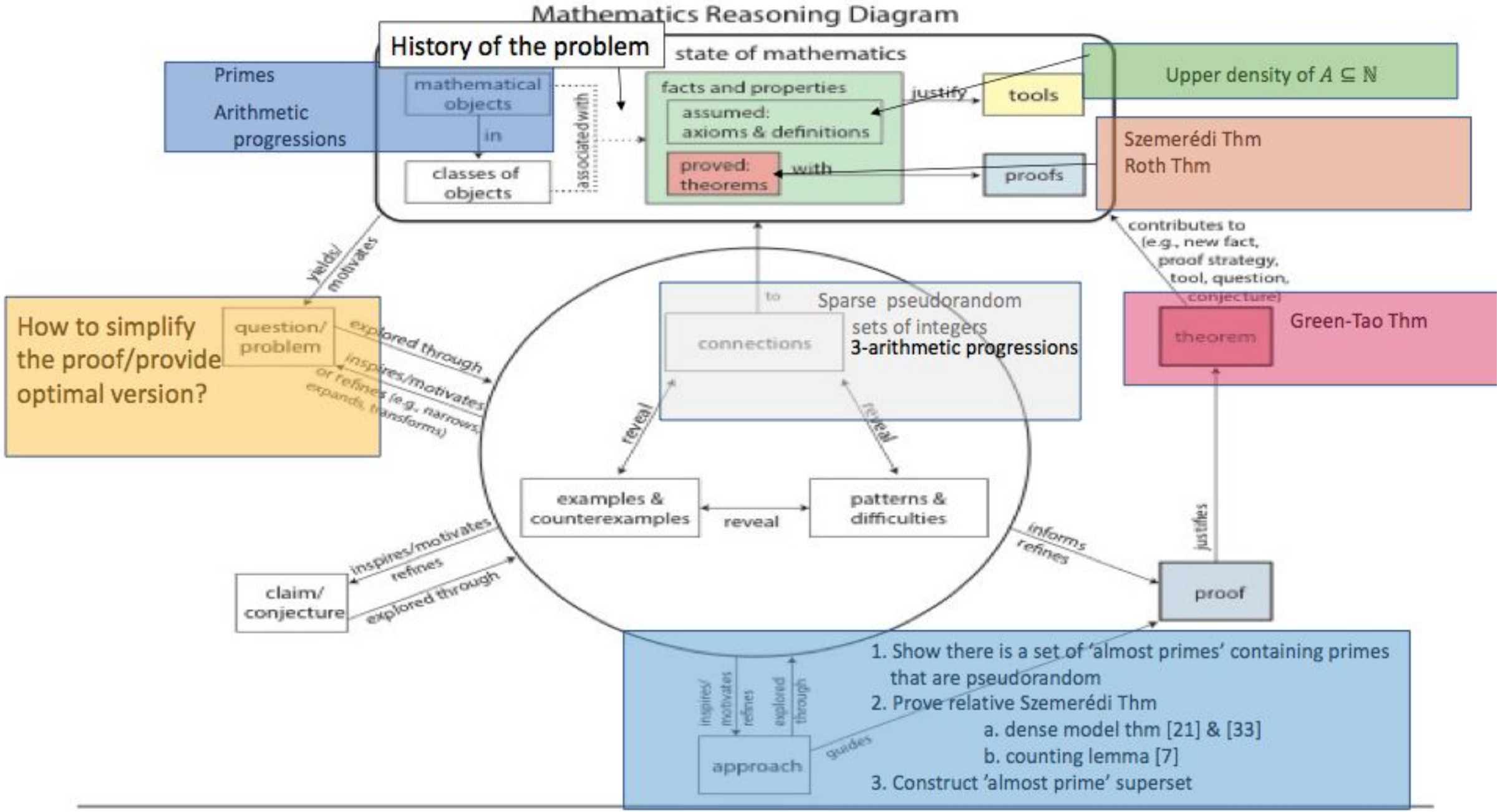
Academic integrity

- MIT takes academic integrity very seriously.
- If there are any such issues with the final version, I will have to report it to MIT.
- Malcah and I are both happy to answer any questions or concerns that you may have at any point during the writing process.

Introductions

What kind of content goes in the intro of a summary (expository) paper?


Mathematics Reasoning Diagram
current state of mathematics
Dirac graph
Hamilton cycle
Random graph
Threshhold p_0
Graphs
Graph properties
mathematical objects
in
classes of objects
associated with
facts and properties
assumed: axioms & definitions
proved: theorems
with
justify
tools
proofs
monotone increasing
Dirac's Thm
(All $G(V,E), with |V| \geq 3, d(G) \geq \frac{n}{2}$ are Hamiltonian)
yields/ motivates
How robust or resilient is a property P of a graph G?
question/ problem
explored through
inspires/motivates or refines (e.g., narrows, expands, transforms)
to
connections
reveal
reveal
examples & counterexamples
reveal
patterns & difficulties
contributes to (e.g., new fact, proof strategy, tool, question, conjecture)
theorem
Extensions on Dirac's Thm & Erdős-Rényi Thm
claim/ conjecture
inspires/motivates
refines
explored through
For n odd,
• 2 complete graphs sharing a vertex, $|V| \geq \frac{n+1}{2}$
• Complete bipartite graph with part size $\frac{n+1}{2}$
inspires/ motivates
refines
explored through
approach
guides
informs refines
justifies
proof


Mathematics Reasoning Diagram
History of the problem
state of mathematics
Primes
Arithmetic progressions
mathematical objects
in
classes of objects
associated with
facts and properties
assumed: axioms & definitions
proved: theorems
with
justify
tools
proofs
Upper density of $A \subseteq \mathbb{N}$
Szemerédi Thm
Roth Thm
contributes to (e.g., new fact, proof strategy, tool, question, conjecture)
theorem
Green-Tao Thm
yields/ motivates
How to simplify the proof/provide optimal version?
question/ problem
explored through
inspires/motivates or refines (e.g., narrows, expands, transforms)
to
Sparse pseudorandom sets of integers
connections
3-arithmetic progressions
reveal
reveal
examples & counterexamples
reveal
patterns & difficulties
inspires/motivates
refines
explored through
claim/ conjecture
informs
refines
justifies
proof
inspires/ motivates
refines
explored through
approach
guides
1. Show there is a set of 'almost primes' containing primes that are pseudorandom
2. Prove relative Szemerédi Thm
a. dense model thm [21] & [33]
b. counting lemma [7]
3. Construct 'almost prime' superset