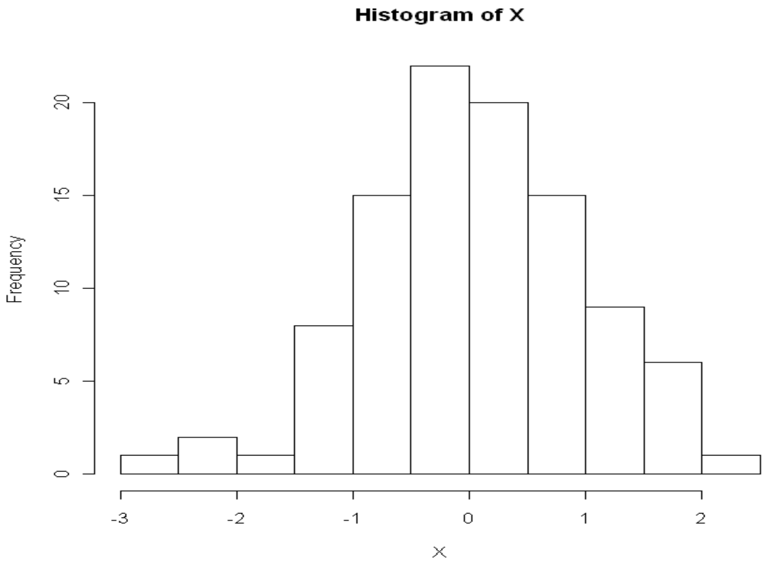
Practice Problems:

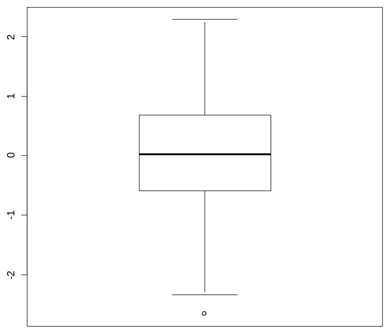
1. Suppose we have the following histogram of some i.i.d. samples



(a) Which of the following distributions would most likely be the distribution of the samples? Normal; Exponential; Uniform; Pareto.

(b) What is a boxplot? What would be the boxplot of the above samples look like?

Solution: (a) Normal. (b) The boxplot:



2. Suppose we have i.i.d. samples from shifted exponential distribution, i.e. the density function is given by $f(x) = e^{-(x-\delta)}$ for $x > \delta$; and 0 otherwise.

(1) Use the first and second order moments in the method of moments to estimate δ.

(2) Find the MLE of δ

Solution:

(1) Since the expectation of shifted exponential distribution is $EX = 1 + \delta$, so if we

use the first order moment, the estimator will be $\hat{\delta}_1 = \frac{X_1 + X_2 + \cdots + X_n}{n} - 1$.

Since the second order moment is $EX^2 = 1 + (1+\delta)^2$, so if we use the second order moment, the estimator will be $\hat{\delta}_2 = \sqrt{\frac{X_1^2 + X_2^2 + \cdots + X_n^2}{n} - 1} - 1$

(2) The MLE is $\hat{\delta}_{MLE} = \min\{X_1, X_2, \cdots, X_n\}$

3. Suppose we have i.i.d. samples $X_1, X_2, \cdots, X_n$ from $N(\mu, \sigma^2)$ distribution.

(1) Assume σ^2 is known, find a level 95% lower confidence bound for μ.

(2) Assume σ^2 is unknown, find a level 95% upper confidence bound for μ.

(3) Suppose we want to test $H_0 : \mu = 0$ against $H_1 : \mu < 0$ using $T = \sum X_i$ as the test statistic. For what values of T should we reject the null hypothesis?

(4) Find the critical region of the level 0.05 test of the above testing problem.

Solution:

(1) Consider $Z = \sqrt{n}\frac{\bar{X}_n - \mu}{\sigma}$, we know that $Z \sim N(0,1)$, so a confidence interval can be constructed based on Z. The level 95% Lower CB is $(\bar{X}_n - 1.64\frac{\sigma}{\sqrt{n}}, \infty)$.

(2) Consider $T = \sqrt{n}\frac{\bar{X}_n - \mu}{S}$, where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$. We know that $T \sim t_{n-1}$. So the level 95% upper CB is $(-\infty, \bar{X}_n + t_{n-1,0.95}\frac{S}{\sqrt{n}})$, where $t_{n-1,0.95}$ is the 0.95 quantile of the t distribution with $n-1$ degrees of freedom.

(3) When T is small enough, we should reject the null hypothesis.

(4) The critical region is $(-\infty, -t_{n-1,0.95}\frac{S}{\sqrt{n}})$.