18.05 Solution 8

1 22.1

(a)

$$\hat{\beta} = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{3\times(1\times 2 + 3\times 1.8 + 5\times 1) - (1+3+5)\times(2+1.8+1)}{3\times(1^2+3^2+5^2) - (1+3+5)^2}$$

$$= -0.25$$

$$\hat{\alpha} = \bar{y}_n - \hat{\beta}\bar{x}_n$$

$$= 1.8 - (-0.25)\times 3$$

$$= 2.35$$

(b)

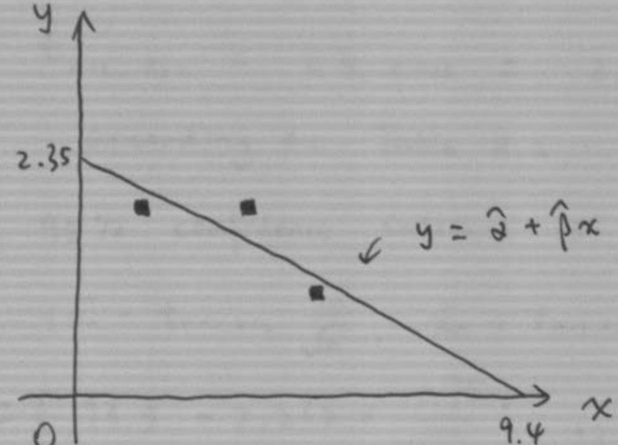
$$r_1 = y_1 - \hat{\alpha} - \hat{\beta}x_i = -0.1$$

$$r_2 = y_2 - \hat{\alpha} - \hat{\beta}x_2 = 0.2$$

$$r_3 = y_3 - \hat{\alpha} - \hat{\beta}x_3 = -0.1$$

$$\Rightarrow \quad r_1 + r_2 + r_3 = 0$$

(c)



2 22.2

$$\hat{\beta} = \frac{4 \times 12.4 - 9 \times 4.8}{4 \times 35 - 9^2} = 0.1085$$

3 22.4

$n = 100$

$$\hat{\beta} = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2} = 2.3848$$

$$\hat{\alpha} = \frac{\sum y_i - \hat{\beta} \sum x_i}{n} = -2.3157$$

4 23.1

$n = 16$

The 95% confidence interval is

$$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \ \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(743 - 1.96 \frac{5}{\sqrt{16}}, \ 743 + 1.96 \frac{5}{\sqrt{16}}\right)$$

$$= (740.55, \ 745.45)$$

5 23.3

$n = 10 \qquad \alpha = 0.05$

$$t_{n-1, \alpha/2} = t_{9, 0.025} = 2.262$$

according to Table B.2, p. 433.

The 95% confidence interval is

$$\left(\bar{x}_n - t_{n-1, \alpha/2} \frac{s_n}{\sqrt{n}}, \ \bar{x}_n + t_{n-1, \alpha/2} \frac{s_n}{\sqrt{n}}\right)$$

$$= \left(93.5 - 2.262 \times \frac{0.75}{\sqrt{10}}, \ 93.5 + 2.262 \times \frac{0.75}{\sqrt{10}}\right)$$

$$= (92.96, \ 94.04)$$

6 23.7

Since $P(2<\mu<3) = P(e^{-3} < e^{-\mu} < e^{-2})$

(e^{-3}, e^{-2}) is a 95% confidence interval for $e^{-\mu}$.