Corrections to the book "Quantum calculus" by Victor Kac and Pokman Cheung

1. page 9, lines 3 and 4 from bottom. Should be n' instead of n
2. page 10, line 11 from bottom. Should be so the formula is true
3. page 11, line 2. Should be $(a - q^2x)$ instead of $(a - q^2a)$
4. page 11, line 3. Should be $(q^{-2}a - x)$ instead of $(q^{-2} - x)$
5. page 11. In the formula above (3.10) instead of $(a - q^{n+1}x)$ should be $(a - q^n x)$
6. page 12. In the example should be $[n]D_q^{j-1}x^{n-1} = [n][n-1]D_q^{j-2}x^{n-2}$
7. page 19, line 6 from bottom. Should be where $m \geq 2$. Consider
8. page 20, lines 8,9. Should be The last line follows from one of the q-Pascal rules (6.3). So (6.5) is true for $0 \leq j \leq m$.
9. page 24, last paragraph. The proof is incorrect, replace it by the following argument: A linear transformation T of rank j is specified by the subspace K of A, mapped by T to 0, and by a collection of j linearly independent vectors b_1,..., b_j in B such that $T(a_i) = b_j$ for a fixed collection of vectors a_1,...,a_j, which together with a basis of K form a basis of A. This proves the desired formula for the jth summand.
10. page 27: The first sentence of Theorem 8.1 should be: Suppose D is a linear operator on the space of formal power series and $P_i = a_i x^i, i = 0, 1, 2, \ldots$ is a sequence, such that all the a_i are non-zero numbers and $D(P_i) = P_{i-1}$ for $i \geq 1$.
11. page 28, lines 5-7: The beginning of the first sentence of the proof should be: We have
12. page 28, lines 5-7.The beginning of the proof of Theorem 8.1 should read: It is easy to see that for any formal power series $f(x)$, we have
13. page 37, line 2: should be if we replace q by $q^{3/2}$ and then put $z =$
14. page 37, last line: should be q^{e_n}th
15. page 38, line 11: should be all partitions of n
16. page 42, in the second line of formulas the symbol $\sum_{m|n}$ should be deleted (twice)
17. page 45, line 10 from the bottom: should be $(1 - q^n)$ instead of $(1 - q^{n-1})$
18. page 52, line 11. Instead of $n \to -\infty$ should be $n \to \infty$

19. page 62, line 7 from bottom. Should be ∞ line 5 from bottom. Should be parentheses around the two summations

20. page 63, line 2. Should be $k, l \geq 1$ in (17.4)

21. page 65. A simple proof of Proposition 18.1: Since $\varphi(x) = \varphi(qx)$, we have $\varphi(x) = \varphi(q^n x)$ for any positive integer n. Tending n to infinity, we obtain $\varphi(x) = \varphi(0)$ for any x.

22. page 66, lines 7,8. Should be: This formula means that $F(u(x))$ is a $q^{1/\beta}$-antiderivative of $f(u(x))D_{q^{1/\beta}}u(x)$.

23. page 74. Corollary 20.1 and its proof should be replaced by the following:

    Corollary 20.1. If $f(x)$ is continuous at $x = 0$, we have for $a, b \in [0, A]$:

    $$\int_a^b D_q f(x) d_q x = f(b) - f(a) \tag{20.2}$$

    Proof. Apply Theorem 20.1 to the function $D_q f(x)$. □

24. page 74. Replace lines 7-9 from the bottom by:

    Now suppose $f(x)$ and $g(x)$ are two functions, which are continuos at $x = 0$. Using the product rule (1.12), we have

    Further, replace lines 4 and 5 from the bottom by:

    We can apply Corollary 20.1 and Theorem 19.1 to obtain

25. page 76. There are divergence problems with the definition (21.6) of the q-gamma function because the function E_q^{-x}, surprisingly, blows up along some sequences as x tends to infinity. (It is because the radius of convergence of the series (9.7) for e_q^x is $1/(1-q)$, not infinity.) However, if we replace the upper limit of the integral (21.6) by $1/(1-q)$, this difficulty is removed, but all arguments on page 77 still hold with little modifications, given below.

26. page 76, line 4 from the bottom. Add: Then we have: $[\infty] = 1/(1-q)$.

    The upper limit of the integral in the definition (21.6) of the function $\Gamma_q(t)$ should be $[\infty]$ instead of ∞

27. page 77. The first sentence should read: First we note that by (9.10), $E_q^0 = 1$ and $E_q^{-[\infty]} = 0$.

    The upper limit of the integral in lines 3 and 7 should be $[\infty]$ instead of ∞.

    The sentence after the definition of the q-beta function should read:

    By the definition of the q-integral (19.7), we have

    In the line that follows the letter a should be removed, the next line should be removed, and in line 9 from the bottom the upper limit of the integral should be 1 instead of ∞. The line after that should be removed.

The upper limit of the integral in line 5 from the bottom should be 1 instead of ∞.

In line 4 from the bottom should be (19.14) instead of (19.15).

In line 3 from the bottom the upper limit should be $[\infty]$ instead of ∞.

28. page 79. Instead of the sentence Then both sides are formal power series in q. should be

    Then both sides are formal power series in two variables q and $v = q^t$.

29. page 80. At the end of the first paragraph add:

    We shall assume that $h > 0$.

30. page 81. In Example replace $(x+b)^N$ by $(x+b)_h^N$ (twice).

31. page 82, line 2. One) should be removed.

32. page 83. In line 6, instead of $D_h(F(x)g(x))$ should be $D_h(f(x)g(x))$

    In line 8 from the bottom, instead of $a < b$ should be $0 \leq a < b$

    In the subsequent definition of $f(x)$ add that $f(0) = 0$

33. page 84. In line 13 after $h > 0$ write and $x > a$. By (22.17),

    In formula (22.19) replace $\frac{}{1n!}|x-a|^{n+1}$ by $\frac{1}{(n+1)!}|(x-a)_h^{n+1}|$

34. page 103. It is not true in general that the polynomials $P_n(x)$ have the form (26.20). Therefore one has to use the following generalization of Theorem 2.1.

    Theorem 26.2 *Let a, q be some numbers, D be a linear operator on the space of polynomials, and $\{P_0(x), P_1(x), \ldots\}$ be a sequence of polynomials, satisfying three conditions:*

    (a) $P_0(a) = 1$, $P_n(a) = 0$ *if n is odd, and* $P_n(qa) = P_n(q^{-1}a) = 0$ *if n is positive even;*

    (b) *deg* $P_n(x) = n$*;*

    (c) $DP_n(x) = P_{n-1}(x)$ *for any* $n \geq 1$ *and* $D(1) = 0$.

    Then for any polynomial $f(x)$ one has:

    $$f(x) = \sum_{n \geq 0 \text{even}} (D^n f)(a)(q^{-n} P_n(qx)) + \sum_{n > 0 \text{odd}} (D^n f)(q^{-1}a) P_n(x)\,.$$

    The proof of this theorem is the same as that of Theorem 2.1. However, unlike Theorem 2.1, Theorem 26.2 can be applied to the operator $D = \tilde{D}_q$ and the polynomials $P_n(x) = (x-a)_{\tilde{q}}^n / [n]^{\sim}!$.

    Moreover, using the same argument as that in the proof of Theorem 20.2, one can derive a similar q-analogue of Taylor's formula with the Cauchy remainder in the symmetric q-calculus.