Remarks added in proof:

1) The conjecture in 3.1 is wrong as Dr. Werner Meyer pointed out: Take arrangements A_1 and A_2 of k_1 and k_2 lines respectively which are in general position to each other. Consider the combined arrangement $A_1 \cup A_2$. Then the numerical characteristics t_r are additive except for t_2 which satisfies

$$t_2(A_1 \cup A_2) = t_2(A_1) + t_2(A_2) + k_1 k_2$$

Let F_1, F_2, F be the quadratic polynomials for $A_1, A_2, A_1 \cup A_2$. Then

$$F(x) = F_1(x) + F_2(x) + k_1 k_2 x^2$$

If we take for A_2 a pencil with k_2 large with respect to k_1, then $F(x)$ becomes indefinite. Maybe the conjecture remains true if one assumes that the arragement "does not contain large pencils".

2) The inequality (3) in 3.1 can be improved by using results of F. Sakai (Semi-Stable Curves on Algebraic Surfaces and Logarithmic Pluricanonical Maps, Math. Ann 254, p. 89–120 (1980)). We have

$$t_2 + \frac{3}{4} t_3 \geq k + t_5 + 2t_6 + 3t_7 + \cdots$$

This is sharp for $A_1(6), A_1(9), A_3^0(3), A_3^0(4)$ and the arrangements with S–T numbers 24 and 25.

3) B. Grünbaum has written to me that the arrangements $A_2(17)$ and $A_7(17)$ of his list are isomorphic.

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Regular Functions on Certain Infinite-dimensional Groups

Victor G. Kac and Dale H. Peterson

To Igor Rostislavovich Shafarevich on his 60th birthday

§0. Introduction

In the paper [18], we began a detailed study of the "smallest" group G associated to a Kac-Moody algebra $\mathfrak{g}(A)$ and of the (in general infinite-dimensional) flag varieties $\mathbf{P}\mathcal{V}_\Lambda$ associated to G. In the present paper we introduce and study the algebra $\mathbf{F}[G]$ of "strongly regular" functions on G. We establish a Peter-Weyl-type decomposition of $\mathbf{F}[G]$ with respect to the natural action of $G \times G$ (Theorem 1) and prove that $\mathbf{F}[G]$ is a unique factorization domain (Theorem 3).

These considerations are intimately related to the study of the algebra $\mathbf{F}[\mathcal{V}_\Lambda]$ of polynomial functions on the variety $\mathcal{V}_\Lambda$ (Theorem 2) and the so-called Bruhat and Birkhoff decompositions of $\mathcal{V}_\Lambda$.

The group G is a (possibly infinite-dimentional) algebraic group in the sense of Shafarevich [20], and belongs to one of the following three classes (we assume A to be indecomposable):

1) Finite type groups. These are connected simply-connected split simple finite-dimensional algebraic groups. In this case almost all the results of the paper are well-known.

2) Affine type groups. Such a G is an $\mathbf{F}^*$-extension of the group of regular maps from $\mathbf{F}^*$ to a group of finite type, or a "twisted" analogue. The simplest flag variety may be regarded as the space of based polynomial loops on a compact Lie group (in the case $\mathbf{F} = \mathbf{C}$).

3) "Wild" type groups. No "concrete" realization of these groups or their flag varieties is known.

The study of the groups G and the varieties $\mathcal{V}_\Lambda$ in the affine case is of particular importance because of applications to topology [2], [8], analysis [1], [9], soliton equations [4], etc.

Throughout the paper the base field $\mathbf{F}$ is of characteristic zero.

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