INTRODUCTION TO ARITHMETIC GEOMETRY
(18.782 FALL 2019)

Course webpage: math.mit.edu/~jwhang/18.782

Instructor: Junho Peter Whang
Email: jwhang@mit.edu

Meeting time: TR 9:30-11 in Room 2-147
Office hours: M 2-4 or by appointment, in Room 2-238A

Prerequisites: 18.702 or equivalent (one year of abstract algebra)
Textbook: There is no required text; lecture notes will be provided.
Exams: There will be no midterm or final exam in this course.

Grading: The final grade will be based on the following scheme.
- Problem sets (60%)
- Final paper (30%)
- Final presentation (10%)

What is 18.782?
This course is an introduction to arithmetic geometry – an incredibly broad area of research at the interface of several disciplines, including number theory and algebraic geometry. Arithmetic geometry has been particularly influential in the study of Diophantine equations (i.e. polynomials with integral coefficients, to be solved in integers or rationals), where the simplest questions are also the hardest to solve. This course introduces some of the basic results and techniques in arithmetic geometry, with emphasis on Diophantine applications. We will aim to understand, sometimes only in special cases, three foundational works from the field around the early twentieth century:
- Hasse-Minkowski theorem and local-global principle for quadrics;
- Mordell’s theorem on the group of rational points on elliptic curves; and
- Siegel’s (and Thue’s) works on finiteness of integral points on affine hyperbolic curves.

Along the way, we will introduce and develop several necessary concepts and theories, including the \( p \)-adic numbers, algebraic geometry of curves, Diophantine approximation, and others.

Problem sets
There will be 6 problem sets, each worth 10% of the final grade. Problem sets and their due dates will be posted on the course webpage. You should submit your problem set to the instructor by email in PDF form (scanned or typed; LaTeX is preferred but not required) on the indicated due date before 11:59 pm.

You are encouraged to collaborate on homework and come to office hours to ask questions, but you are required to write up your own solutions. You should name all collaborators or other sources of information on each assignment. Problem sets submitted after solutions are posted will not be accepted. To request an extension on a problem set, you should write to the instructor prior to the deadline, together with a documentation letter from S3.
Final project
Each student will write a short expository paper (approx. 5-10 pages) on a topic of their choice in arithmetic geometry, and give a short presentation (approx. 30 minutes) on the topic at the end of the semester. The exposition should be aimed at fellow students learning the subject. Submitted articles will be compiled and made available to the class at the end of the semester.

An initial list of suggested topics can be found on the course webpage. You are encouraged to meet with me early on in the semester to ask about the individual topics, my expectations on them, and references. Most of the topics will require reading and digesting a short paper or a section in an appropriate textbook, which will be secured for you if necessary. Each topic can be chosen by at most one student on a first-come, first-served basis. If you want to pursue a topic that falls within the scope of the course but is not listed on the course website, you should discuss it with me first.

Here is a tentative timeline for the final project; the exact dates may change.

- Week of October 21 or earlier: meet with instructor and decide on a topic.
- Week of November 25: discuss/rehearse presentation with instructor.
- December 3: final paper due.
- December 5 and 10: presentation days.

Reading and writing a mathematical paper, expository or research, take time and practice. You are encouraged to meet with me frequently for guidance throughout the process.

Syllabus
The following is a tentative list of planned topics for the course.
The topics may change as the semester progresses.

(a) Local-to-global principle for conics
- Normed fields and completion, construction of \( \mathbb{Q}_p \), Ostrowski’s theorem
- Inverse systems, \( p \)-adic integers and their properties
- Completeness of \( \mathbb{Q}_p \), Hensel’s lemma
- Squares in \( \mathbb{Q}_p \), field extensions, algebraic closures
- Quadratic forms
- Hasse-Minkowski theorem, rational points on conics

(b) Algebraic geometry of curves
- Affine space, affine varieties, irreducibility, dimension
- Projective space, projective varieties, affine charts
- Morphisms, function fields, rational maps
- Hypersurfaces, smoothness, tangent space
- Curves and their function fields, valuations
- Divisors, Picard group
- Riemann-Roch theorem for curves

(c) Arithmetic of curves
- Elliptic curves, Weierstrass equation, group law
- Mordell’s theorem
- Height functions
- Diophantine approximation
- Thue’s theorem (and Siegel’s theorem)
**Student Support Services**
If you are dealing with a personal or medical issue that is impacting your ability to attend class, complete work, or take an exam, please discuss this with Student Support Services (S3). The deans in S3 will verify your situation, and then discuss with you how to address the missed work. Students will not be excused from coursework without verification from Student Support Services. You may consult with Student Support Services in 5-104 or at 617-253-4861. Also, S3 has walk-in hours Monday-Friday, 10-11am and 2-3pm.

**Disability and Access Services (DAS)**
MIT is committed to the principle of equal access. Students who need disability accommodations are encouraged to speak with Kathleen Monagle, Associate Dean, prior to or early in the semester so that accommodation requests can be evaluated and addressed in a timely fashion. Even if you are not planning to use accommodations, it is recommended that you meet with DAS staff to familiarize yourself with the services and resources of the office. You may also consult with Disability and Access Services in 5-104 or at 617-253-1674. If you have already been approved for accommodations, please contact me early in the semester so that we can work together to get your accommodation logistics in place.