Problem Set 8: Models, compactifications, identifying spaces, characterizing subspaces

Your name:

Due: Tuesday, April 26

Read the notes on the course website related to adjunctions, compactifications, and the compact-open topology (sections 1–8).

Problem 1 (8)**.** Have you done the reading?

Problem 2 (B8)**.** Let X be a compact space and Y be a metric space. Prove that the metric topology on the set of continuous functions $\hom(X, Y)$ (defined by the metric $d(f,g) = \sup_{x \in X}\{d_Y(f(x), g(x))\}$) is the same as the compact open topology.

Problem 3 (8)**.** Give a model for *the space of rays starting at the origin in* $\mathbb{R}^n$ and prove that it is homeomorphic to S^{n-1}.

Problem 4 (16)**.** Prove that $\mathbb{RP}^2$ is not homeomorphic to $\mathbb{D}^2$.

Problem 5 (B8)**.** Consider two points, α and β, in the mapping space $\hom(S^1, \mathbb{R}^2 \setminus \{0\})$.

a. Describe geometrically what it would mean for there to be a path from α to β in this space.

b. Give an example of two points that are in the same path component of this space.

c. Give an example of two points that are not in the same path component.

Problem 6 (8)**.** Let $f : S^n \to \mathbb{R}$ be a continuous function. For any point $x \in S^n$, denote by τx the antipodal point to x. Prove that there is at least one point $x \in S^n$ so that $f(x) = f(\tau x)$.

Problem 7 (24)**.** This problem deals with a variant of $\mathbb{RP}^2$.

a. Give a model for the *space of two distinct lines passing through the origin in* $\mathbb{R}^2$—I will call this space L.

b. Prove that L is not compact, and describe a compactification Y for this space.

c. What does the space $Y \setminus L$ look like?

d. When does a continuous function $L \to \mathbb{R}$ extend to a continuous function $Y \to \mathbb{R}$?