Problem Set 6: Compactness and The Zariski topology

Your name:

Due: Thursday, April 7

Read the notes on the Zariski topology posted on the web page.

Problem 1 (8)**.** Did you do your reading assignment?

Problem 2 (24)**.**

a. Let R be a commutative ring with unit ($1 \in R$). Prove that $\operatorname{spec}(R)$ is compact.

b. When is $\operatorname{spec}(R)$ Hausdorff?

Consider the map

$$i : \mathbb{R}^n \to \operatorname{spec}(\mathbb{R}[x_1, \ldots, x_n])$$

that is defined by taking the n-tuple $(r_1, \ldots, r_n)$ to the ideal generated by the degree one polynomials

$$x_1 - r_1, \ldots, x_n - r_n.$$

c. Prove that the map i is injective. Identifying $\mathbb{R}^n$ with its image under the map, it acquires a topology as a subspace of $\operatorname{spec}(\mathbb{R}[x_1, \ldots, x_n])$. Denote $\mathbb{R}^n$ with this topology by $\mathbb{A}^n$

d. What is the closure of $\left\{(n, \frac{1}{n}) : n \in \mathbb{N} \setminus \{0\}\right\} \subset \mathbb{A}^n$?