

# Chapter 1

## Computer manipulatives and student engagement in an online mathematics course

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**Abstract** The MIT Mathlets constitute a collection of interactive JavaScript visualizations. They have been used in lecture and homework at MIT since 2002. Beginning in 2015 many of them have been integrated into several MITx courses, for both MOOC and residential use. We will describe these tools, how they have been modified and extended for integration into an online environment, and the student reaction to them. We also offer a simple model accounting for the distribution of learner persistence.

### 1.1 Introduction

Two trends dominate discussion of contemporary university level STEM education: the use of online resources, and the introduction of active learning methods. Are they in conflict with each other?

A popular typography of active learning strategies is provided by Michelene Chi's ICAP framework [1]. Chi describes four types of student activities, which we briefly sketch in relation to the courses we study. "Passive activities" are things like attending a traditional lecture, or, in an online setting, watching a video. The act of moving a slider in a Mathlet is an example of "Active" student behavior, but making connections between various representations of the effect of moving

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the slider qualifies as “Constructive.” This framework terms activities that involve collaboration among peers as “Interactive,” and posits that they are the most potent in producing cognitive engagement and hence optimizing learning. Normally this happens in a classroom setting, with students interacting directly with each other. One of our research questions is this: Can the power of this type of activity be reproduced in an online setting, when learners typically interact only with course material, or, at best, through rather formalized online fora?

In this article we discuss the use of one online technology to encourage student engagement in an MITx mathematics course, namely the MIT Mathlets supported by structured activities and an online forum.

We begin in Section 1.2 by introducing the MIT Mathlets: their history, philosophy, and usage. Next we will review the principles shaping the creation of courseware built on the edX platform, for use not only in an open online course but also residually in the courses as offered at MIT. In Section 1.3 we introduce the edX platform and the development of mathematics courses based on it, and describe how the Mathlets were integrated into that framework. We provide three illustrations, along with examples of student responses to this material. Section 1.5 compares learner usage of problems based on Mathlets with non-Mathlet based problems. Finally in Section 1.6 we suggest a primitive mathematical model that attempts to account for the variation in persistence observed in these courses and presumably others.

Three mathematics courses have been hosted by MITx: 18.01 Calculus I, 18.03 Ordinary Differential Equations, and 18.05 Probability and Statistics. We choose, in this paper, to focus on 18.03. The data for 18.03 are diverse and extensive, and what we have seen in the other two courses does not appear to offer further insights.

We end this introduction with a very brief review of some of the relevant literature.

The interesting study [2] presents evidence of the increase in both learning gains and learner persistence resulting from use of online activities (as created by CMU’s Open Learning Initiative) in comparison with text and video presentation.

Learner persistence is of serious interest to MOOC operators, and careful work on prediction of dropout profiles include [3]. Our work deals with persistence, but we use a different indicator, studying the total number of problems tried or attempted by each individual student over the entire duration of the course.

There is a body of research around the use of discussion boards in online courses. The initial study reported in [4], for example, suggests that there is a positive correlation between the level in Bloom’s taxonomy of a question posed on a discussion board and student participation. The use of the edX forum feature in the courses we discuss was passive; students were encouraged to discuss problems on it, but it was not used as a place in which questions were asked. We are also, in this study, interested in the use of discussion boards primarily to gain information about student attitudes toward various problem types and to understand how students used them as interactive enhancements of their response to problem assignments.

Very little formal research seems to have been done on the relative efficacy of the kind of “computer manipulative” represented by the MIT Mathlets in student learning gains. An early study of student reaction to graphical methods appears in [5], and a careful study of student reactions to the MIT Mathlets themselves is presented

in [6]. We do not address this issue at all in this study, focusing instead on learner persistence measured by the total number of problems viewed or solved.

## 1.2 The MIT Mathlets

The MIT Mathlets form a suite of interactive simulations, or, with a nod to Montessori learning tools, “manipulatives,” freely available today in JavaScript at <http://mathlets.org> under a Creative Commons BY-NC-SA license. They have been used at MIT since 2002 [6]. They are designed to resolve several tensions present in university level mathematics education:

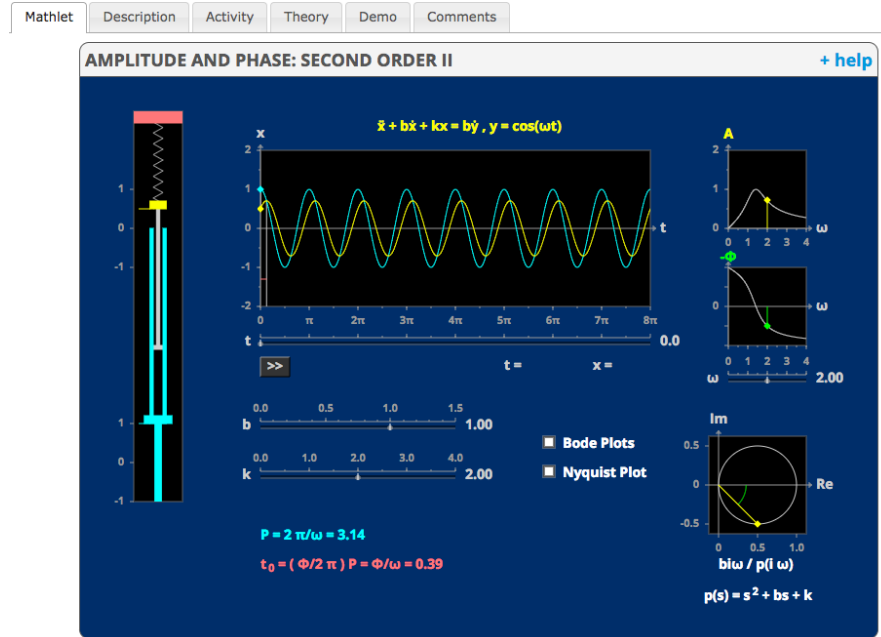
1. Context vs generality. Many of the Mathlets show a mechanism or electrical circuit, along with the differential equation modeling it and graphs showing the input and system response.
2. Example vs theory. At the blackboard, a teacher is often faced with the choice between doing a general case, with symbolic constants, or a special case, giving the constants specific values. The slider-based highly responsive character of the Mathlets allows the student to visualize entire families of special cases, in effect filling out the general case.
3. Algorithm vs theory. The theory underlying various algorithms is often most clearly conveyed graphically. For example, the use of complex numbers in understanding linear time-invariant systems becomes less mysterious and more significant when the “Nyquist plot” can be visualized.

Work on the MIT Mathlets began in the fall of 2000 under the leadership of the second author, thanks to a grant from the MIT d’Arbelloff Fund for Excellence. Inspiration was derived from an earlier project “Interactive Differential Equations,”<sup>1</sup> and the lead programmer from that project, Hu Hohn of the Massachusetts College of Art and Design, formed a key part of the team and did all of the programming. The MIT Mathlets were incorporated into the standard undergraduate ordinary differential equations course, 18.03, starting in the spring of 2002, both in lecture and as the basis for homework assignments. At first they were presented as UNIX executables and students were required to go to computer clusters to use them. Student useage and attitude was studied by Deborah Upton during the 2002–2003 academic year and reported on in [6]. This formative assessment resulted in improvements of the applets and in much better understanding of how to use them effectively. Between spring 2002 and fall 2005 an extensive Java library was written, chiefly by Jean-Michel Claus, the Mathlets were thenceforth available on the web.

With the deprecation of Java, it became necessary to port this material into JavaScript. This process was started in fall 2011. This was a time consuming and expensive project, underwritten by a grant from the MIT Office of Digital Learning and overseen by Jean-Michel Claus with contributions from Brandon DeRosier and

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<sup>1</sup> <http://ptraci.math.hmc.edu/software/interactive-differential-equations-ide>.



**Fig. 1.1** Initial state of a Mathlet from mathlets.org, with Bode and Nyquist Plot windows toggled shut

Heather Petrow. The first batch of JavaScript Mathlets was published in fall 2014 and completed in January 2016. Heather Petrow also made the Mathlets accessible on a cell phone.

Over the years the original collection of “manipulatives” was extended to cover topics from a number of other subjects: Calculus Mathlets were designed with the help of Heidi Burgiel and integrated into an MIT OpenCourseWare Scholar course.<sup>2</sup> As part of the creation of a new Probability and Statistics curriculum,<sup>3</sup> supported by a grant from the Davis Educational Foundation, a suite of Mathlets focused on these topics were designed by Jeremy Orloff. Further Mathlets have been created in support of courses in the MIT Departments of Physics, Aero-Astro, Mechanical Engineering, and Health Sciences and Technology [7]. All are freely available through a Creative Commons license on the MIT Mathlets website <http://mathlets.org>. Still other Mathlets were written by the first author specifically for use in the MITx course 18.01x.

The construction of the MIT Mathlets has been based on the following design principles. Each Mathlet

1. is narrowly focused on a single objective

<sup>2</sup> <https://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010>

<sup>3</sup> <https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014>

2. represents the same information simultaneously in several different ways, linked by color coding
3. requires the student to select settings, and responds to these settings in real time
4. displays information progressively, if appropriate, rather than all at once, by use of toggles
5. is easy to use, based on uniform conventions
6. relies on scrupulously correct mathematics
7. reflects analytic methods the student is expected to execute by hand
8. is carefully designed to be visually attractive
9. is accompanied by a brief “Help” page that simply outlines the Mathlet functionalities
10. works with keyboard shortcuts and screen readers, following a lly guidelines.

The Mathlets have been posted on the WordPress website [mathlets.org](http://mathlets.org) since June, 2009. Below is a screenshot of a Google Analytics Dashboard summary of daily Session at [mathlets.org](http://mathlets.org) over the 90 days preceding January 12, 2019. It shows several features characteristic of the usage of the Mathlet website:

- During the term, there is a very strong weekly oscillation around an average rate of about 250 sessions per day.
- The amplitude of this oscillation is about 100 . . .
- and the phase is such that maxima always occur near to midweek and minima always occur over the weekend.
- When classes are not in session, there is a background usage that does not show much structure.
- Occasionally there is a dramatic spike in hits. This has always been explained by some twitter feed linking to the Mathlets website.

The spike shown here is unusual in the slowness of the falloff of interest following the spike. These later peaks show the weekly oscillation, and may in fact be unrelated to the Twitter spike but rather reflect students preparing for exams using the website.

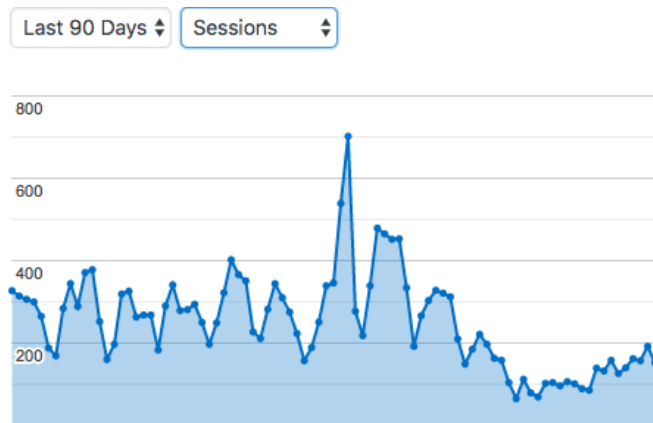
### 1.3 The introduction of the edX platform

Around the same time that the project to translate the Mathlets into JavaScript began, MIT announced the founding of MITx, a company that was soon to create an interactive learning platform to provide free higher education courses to the world for free.<sup>4</sup> This company became edX with the joining of cofounder Harvard, and by now has scores of affiliated universities and other organizations.

In time MITx became an office under the MIT Vice Provost for Open Learning with the explicit goal of supporting MIT faculty teams to produce digital learning

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<sup>4</sup> <http://news.mit.edu/2011/mitx-education-initiative-1219>



**Fig. 1.2** Daily session usage of mathlets.org

content available on the open edX platform. This content is used globally, as Massively Open Online Courses (MOOCs), and locally, in support of MIT's residential education program.

Two courses from the Mathematics Department were chosen for development within the edX platform: 18.01 Calculus, a single variable calculus course required of all MIT undergraduates, and 18.03 Differential Equations, a course taken by over 750 MIT undergraduate students, typically freshman and sophomores, every academic year. These residential courses are organized into three one hour lectures with all students lead by professors, and two one hour recitations of around twenty students each week led by graduate student Teaching Assistants.

Several factors suggested these courses:

1. The MIT treatment of single variable Calculus differed in important ways from any textbook treatment, and the advent of MITx was perceived as a chance to provide students with material more closely matching the actual goals of the course than any existing textbook. Moreover the MIT educational approach has always been centered on solving problems, and the MITx framework offered the chance to provide students with immediate feedback on their proposed solutions.
2. The curriculum for 18.03 Differential Equations was being revised to include more linear algebra, and remove the Laplace transform. The revision meant that there was no one textbook that covered the content in the course. The course already required students to go to the MIT Mathlets website and interact with one or another of the Mathlets in almost every homework assignment, so it was natural to envision embedding individual Mathlets into an online text.
3. The material on Laplace transform, impulses, convolution, and transfer functions that had been removed from the course formed the basis for a new course 18.031 System Functions and the Laplace Transform. The Mathlets play a big role in this course too. The enrollment in this course is much smaller – on the order of twenty students – and the mode of instruction in use was highly interactive [1],

with students working on the board together for most of the class time on both active and constructive problems. The open edX platform offered the opportunity to complete the transformation of the course into a flipped classroom, with all the material presented online and the class time more completely devoted to working with students on problems.

Thus the goal in using the edX platform for both courses was to

- Improve on the textbook with a much enhanced digital version
- Embed Mathlets into the digital text with associated activities
- Incorporate active and constructive [1] learning by having retrieval problems, practice problems, modeling problems, and faded worked examples [8] appropriately paced throughout the text
- Improve sustainability (for future teachers of the subject)
- Provide a reference for former students in subsequent years
- Outreach to the rest of the world by making our versions available for free globally as MOOCs

### 1.3.1 The design of active learning digital content

The first set active mathematics MOOCs created and released between 2014 and 2016 were three modules of single variable calculus:

- Calculus 1A: Differentiation,
- Calculus 1B: Integration,
- Calculus 1C: Coordinate Systems and Infinite Series.

The naming of these modules was intended to be suggestive of the AP Calculus curriculum — Calculus 1A and 1B together comprise the Calculus AB curriculum, and all three modules together comprise the Calculus BC curriculum. In total, we created 46 active digital lectures. Each of these lectures was comprised of on average 9 short video segments, 12 small text pages, and 23 computer graded formative problems with immediate feedback and full solutions. There were 11 homework assignments, each divided into a “Part A,” consisting of problems of a mainly computational character and testing only one or two concepts in each problem, and “Part B” problems that were typically problems of an applied or exploratory character that combined many ideas and themes from throughout the course. There were 3 summative exams, one for each module.

The problems (from lecture or homework) could be multiple choice, choose all that apply, numerical entry, formula entry, custom graded problems such as integration set-up, or summation notation set-up problems, sketchResponse problems [9], where students could draw with a mouse or touch pad and computer graders give automated feedback. The Mathlets were integrated into lecture content mainly through problems which would help to illustrate concepts. These Mathlet problems could be of one or more of the above problem types, or they could be graded by grading the state of

sliders that the student placed in manipulating the Mathlet itself. In all three modules, we used 17 Mathlets, of which 7 were created specifically for these MOOCs. We only used the grading of the student state of the Mathlet in a few instances where the Mathlets were designed specifically to be graded in this way. Most Mathlets are designed to illustrate concepts (such as the geometric definition of the derivative as the slope of a tangent line obtained as a limit of slopes of secant lines) through manipulation of the sliders and points on the Mathlet itself, not as a static state of the Mathlet with specific numerical readouts and positions. In these cases, we found multiple choice questions external to the Mathlet to be particularly useful in that we could ask students to perform certain actions, and ask them their observations, while at the same time limiting the space of observations to the selected set of options given in the multiple choice question.

The use of the Mathlets in Calculus served as a test of what was possible, and to see how well they were received. Overall, students seem to appreciate these interactive visualizations as shown from a post from the last run of Calculus 1A with regard to the very first Mathlet observed in Calculus 1A: Differentiation:

Amazing MIT's amazing Mathlet

Wow! I am just truly amazed by the visualization of the tangent via Mathlet. Oh my God! It's just really mesmerizing to observe the tangent at a point at this level of zooming. Wow MIT, I Thank You a lot for this kind of visualization. . . .

We continued to embed the Mathlets into active digital content for 18.03 Differential Equations and 18.031 System Functions and the Laplace Transform between 2016 and 2018. The vast majority of the Mathlets were created for these courses, so we relied on them much more heavily in activities. The residential content for 18.03 Differential Equations was broken into four modules, each released as a MOOC:

- Introduction to Differential Equations
- Differential Equations:  $2 \times 2$  Systems
- Differential Equations: Linear Algebra and  $n \times n$  Systems of Differential Equations
- Differential Equations: Fourier Series and Partial Differential Equations

These modules consisted of 42 active learning lectures and recitations containing 35 Mathlets. On average each lecture contained 5.5 short videos, 15 short text blocks, and 10 automatically graded problems with immediate feedback and full solutions. There were 15 homework assignments, each containing a Part A and a Part B as in the Calculus course. The part B problems were scaffolded by a new addition, the recitation exercises. Each lecture had a paired "recitation," which consisted of a series of problems designed to encourage collaboration by students on the Forum. No solutions were posted, though we monitored the forum and corrected erroneous claims. This was also an attempt at adding collaborative and interactive [1] learning into the active and sometimes constructive learning framework of our online course, which was typically completed by students alone. We created these recitations after feedback from our Calculus MOOCs suggested students felt the leap between lecture content and part B homework was too great. This also made the MOOC more similar to the structure of the on-campus class which provides small groupwork sections



following a faded worked-example model [10, 8, 11, 12] to scaffold and prepare students for the Part B homework.

The module 18.03L System Functions and the Laplace Transform was run residentially as an active learning class. It was released as a MOOC titled

- Transfer Functions and the Laplace Transform.

This module consists of 12 active learning mini-lectures containing 7 Mathlets. On average each lecture contains 4 short videos, 15 short text blocks, and 15 automatically graded problems with immediate feedback and full solutions. Some of these problems were weighted more heavily and were based on the more difficult activities done in class as groupwork.

## 1.4 Integration of Mathlets in an online setting

The creation of online material offered the opportunity to enhance one of the objectives of the redesign of the residential Differential Equations course, namely an increased focus on mathematical modeling. An effort was made to create authentic experiences with the application of the methods of the course. Real world situations were presented through lecture and video, with data captured from actual experiments, and appropriate mathematical models were developed. The Mathlets allowed the students to conduct virtual experiments as part of this learning sequence. We present three examples, taken from the online Differential Equations course. The first example develops the response of a series RLC circuit to sinusoidal input signals, with emphasis on the importance of specifying what constitutes the system response. The other two come from System Functions and the Laplace Transform.

### 1.4.1 System response and resonance in a series RLC circuit

An example of one applied experience is in Lecture 11 Resonance of the MOOC Introduction to Differential Equations. In this section of the lecture, a series RLC circuit forms the basis of a worked example used to explore resonance. This example illustrates how one system can be interpreted differently based on what is considered to be the system response, and how the resonant behavior depends upon that choice. The Mathlet used in this sequence, Series RLC Circuit, was developed in collaboration with Peter Dourmashkin of the MIT Physics Department. Here is a listing of the components of this section.

1. Instructor led video example of setting up a model
2. Student problems to find related models and compare (self-reflection)
3. Demo/laboratory video showing frequency response for voltage drop across a resistor
4. Mathematical exploration – identify the resonant peak

5. Mathlet visualization tool used to elicit a prediction [13] by the student (of how resonant peak changes if resistance is increased)
6. Demo video to confirm or correct this prediction
7. Demo video of frequency response for voltage drop across capacitor and inductor (student modeled scenarios)
8. Predict behavior for voltage drop across capacitor and inductor based on mathematics
9. Activity to use Mathlet visualization of circuit to confirm mathematics

#### **1.4.2 Using a Mathlet to illustrate Bode plots**

The goal here is to help students move to the next step in their study of frequency response, namely the interpretation of Bode plots. To begin with, the example of the series RLC circuit is recalled as a model of a passive AM radio receiver. This circuit is regarded as a type of filter that responds preferentially to certain input frequencies and not others. The student is then asked to use the Mathlet Amplitude and Phase: Second Order II, that displays a mechanical system controlled by the same mathematics as the RLC circuit example. The student explores how the system response changes based on the input frequency, both by looking at the response, and also at the Bode Plot (which was suppressed in prior usages of this same Mathlet).

Students are asked to answer questions about the value and sign of the phase lag near the resonant frequency, and are asked to choose all descriptors that apply in qualitatively explaining how the Bode plot changes when one of the parameters in the model is changed. Because the Mathlet models a mechanical system, students are then asked a follow-up question to translate the answers from their exploration into what parameters in the RLC circuit must be adjusted to achieve the same effect.

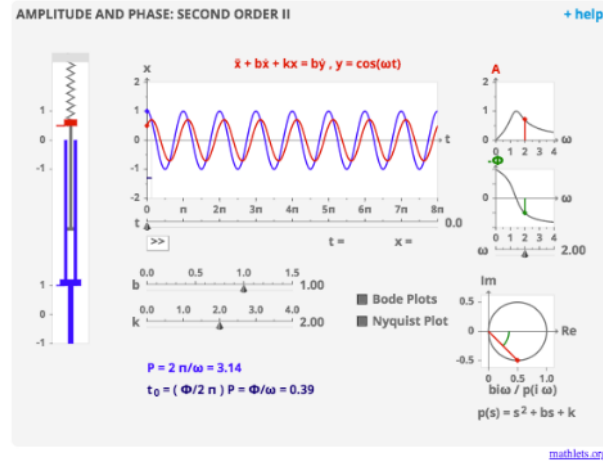
The culmination of this sequence, and indeed of the course, is a re-introduction (via a video segment) of an actual mechanical artifact, built by MIT Mechanical Engineering Professor David Trumper: a very nice sinusoidally driven damped coupled oscillator. The student was shown the gain Bode plot and asked to match it by placing poles and zeros in the Mathlet. In this last lecture, we explain that this system with the second mass removed, but considering the voice coil actuator in the model gives rise to a transfer function with three poles. Students were given a completely optional activity to attempt to recreate the Bode plot from the data of this actual demonstration using the Mathlet. It was exciting to see the intensity of students interaction on the discussion forum regarding this optional activity — sharing subtleties and solutions, including screen shots of the Mathlet.

## Bode Plots

3/3 points (graded)

Each of the Amplitude and Phase Mathlets allows visualization of plots of the gain and the phase lag as functions of frequency. To see these two graphs on the Mathlet below, check the box marked "Bode plots." (Why "Bode"? Take a minute now to understand how the Bode plots reflect the behavior of the system. Note that the lower graph plots the phase gain rather than the lag, that is,  $-\phi$  rather than  $\phi$ .)

Here are some questions for you to guide your exploration of the Mathlet. Remember, you can change the system parameters  $k$  and  $b$  using the sliders below the graph. Vary them and confirm your hypothesis that  $\omega_p = \sqrt{k}$ .



(Answer these questions based on your exploration of the Mathlet.)

**Fig. 1.3** The same Mathlet as shown in Figure 1.1 as incorporated into an MITx course page, with Bode and Nyquist plots selected. Note that the color scheme of the Mathlets used on the open edX platform was changed from the original blue to a muted grey. The new color scheme was chosen so that when embedding the Mathlets as iframes into the text or problem, the Mathlet feels integrated seamlessly with the rest of the content on the platform. This new color scheme was also carefully chosen to meet accessibility guidelines.

### 1.4.3 Using a Mathlet to give meaning to the convolution integral

The convolution integral is introduced and defined through a worked example that illustrates where each term comes from, what it means, and why it solves a particular differential equation. To illustrate convolution as an operation, the Mathlet Convolution: Accumulation is employed, supported by an activity. First the student is instructed to perform a series of operations with the Mathlet, which allow her to walk through all of the visualizations and details of the Mathlet. The student is asked to describe what a particular shaded region represents, and they answer a multiple choice question which is both leading, but also follows the natural expert thought pattern one might expect. Then, the second author narrates a short voiceover video that helps to explain how the visualizations connect to the context of the convolution example presented previously. Next, the student uses the Mathlet to observe a behavior in an abstract example that allows them to connect the visual understanding of convolution to an analytic answer. Finally, a new example with a new context

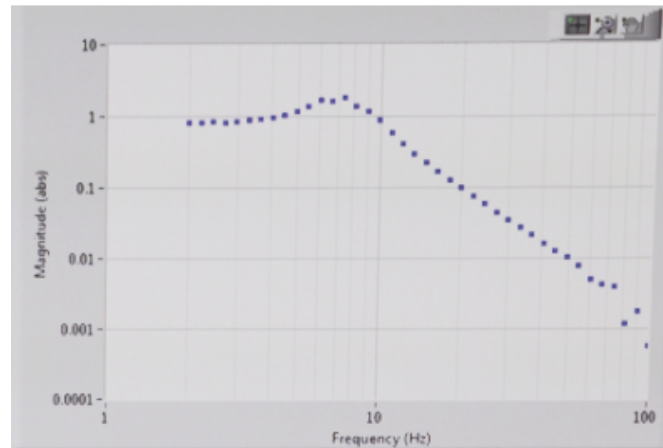


Fig. 1.4 Data from the demonstration to be replicated

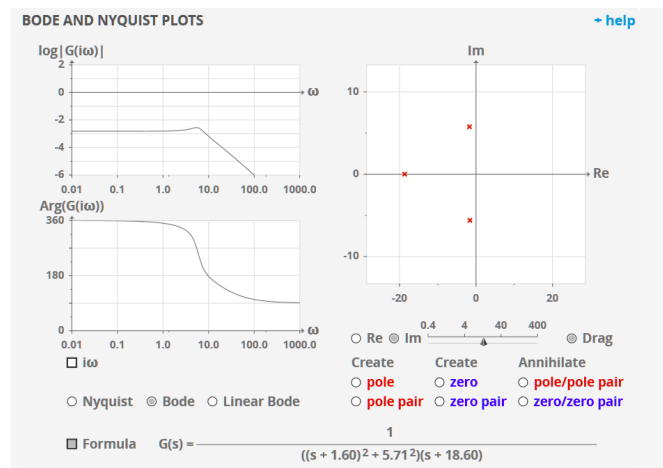
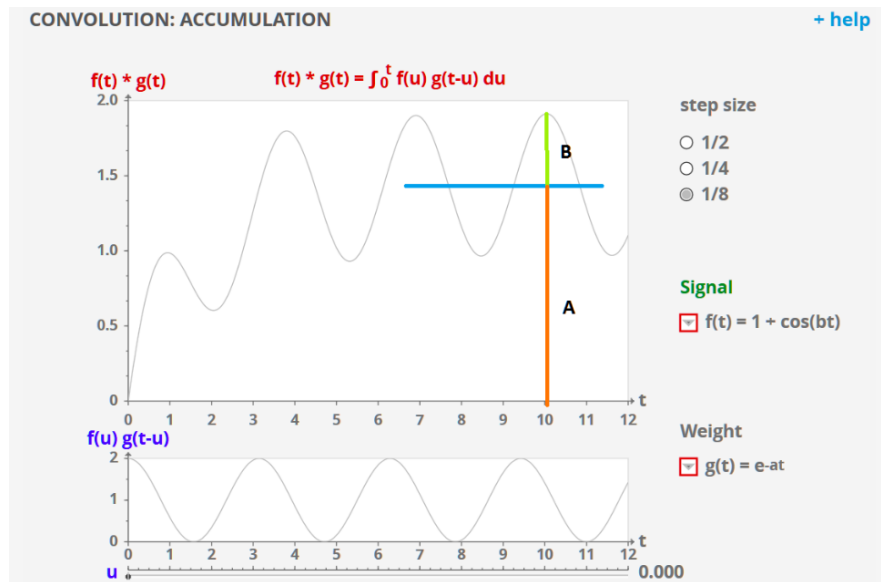


Fig. 1.5 Student Mathlet settings posted on the forum

is presented. The student first works through the analytic answer and then uses the Mathlet at the end to compare the analytic answer with the visualization to reflect on and understand how the mathematics worked.

In this problem also, we saw the students actively engaging with each other on the forum to understand how to use the Mathlet to see the solution as suggested by the problem statement, and helping each other out by posting screen shots of the Mathlet with annotations. We also see student reflection on the purpose of the activity and how they are suppose to use the Mathlet and new ideas of convolution compared to older ideas from earlier in the course.



**Fig. 1.6** Student forum post: a student imposed the colored lines on a screenshot of the Convolution: Accumulation Mathlet to helping another student see how the steady state solution appears to be a sinusoid shifted of amplitude  $B$  shifted vertically by a number  $A$ .

Here's what I took from the problem. When a new idea is introduced, it's useful to show how it fits in with previous knowledge. The mathlet is driven by the details of convolution and displays a response with a transient, and a steady state that looks like a constant value upon which a sinusoid has been imposed. The review problem lets us verify that the old methods agree that this is the case (and gives us the actual values for the steady state coefficients). This is also an example of remembering the old pieces while new pieces are being collected, and observing how new and old fit together. 7.6 Activity: radioactive material continues to gather and consolidate.

These discussions and reflections tying new content to the older methods was exactly the points of these incorporated Mathlets, and it was refreshing to see these discussions occur in the forum.

## 1.5 Comparison of usage

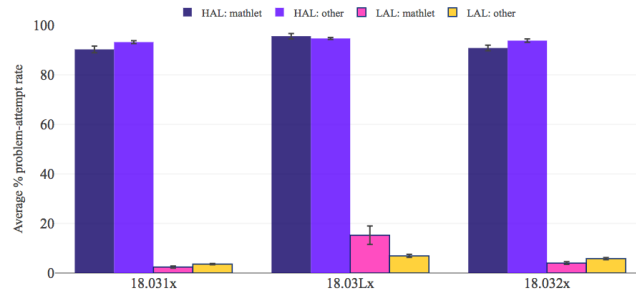
In this section we discuss what can be discovered about the usage of the MIT Mathlets from data collected by the edX platform. The data consisted of numbers of views and attempts for each of the problems in the various courses, identified to student. Our basic question was this:

How did usage of the Mathlets vary from one module to another and across different classes of users?

For example, did they induce less motivated students to stick with the course longer? Did the less motivated group use them more or less (relative to problems not involving the Mathlets) than the more motivated group? Did the frequency of views and attempts of Mathlet based problems vary significantly between the various modules?

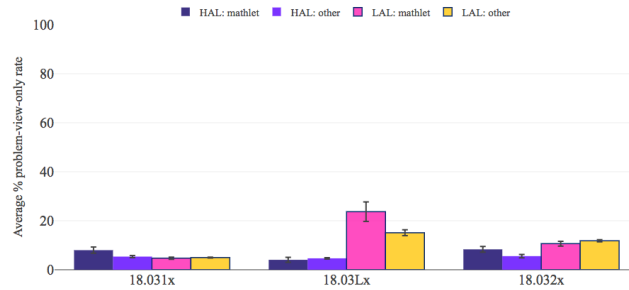
To study the usage of the Mathlet and non-Mathlet problems we divide the group of registered users into classes. As is well-known, many individuals register for online courses but are entirely passive. In order to exclude that group, we define as "active learners" (AL) all registered users who attempted at least one problem over the entire module. We distinguish a subset of these active learners as "high-activity learners" (HAL), defined arbitrarily as those learners who are in the upper 40 percentile by the total number of problems attempted. (Some problems allowed multiple tries; we count these as a single attempt.) Our overall conclusions about usage do not change much based on our choice of the cutoff percentile in the range between 20–60 percentile. The remaining active learners are "low-activity learners" (LAL).

Figure 1.7 displays the percentage of Mathlet problems and of non-Mathlet problems attempted by the average student in each of the two classes of Active Learners. Equivalently, this is the percentage of learners in each of these classes who attempted a given Mathlet or non-Mathlet problem, on average. The error bars indicate the standard error based on the first interpretation. Figure 1.8 shows the same data but using views only (without attempts).



**Fig. 1.7** Problem attempts

These data show that students of both classes generally attempted Mathlet and non-Mathlet problems at about the same frequency. An exception to this general rule occurs in the much larger fraction of LAL attempts of Mathlet problems than non-Mathlet problems in 18.03L. We were excited by the strength of this signal, but in the end we conclude that it has a mundane explanation: Problems involving Mathlets were introduced much earlier in 18.03L than in either of the other modules, so the number of LAL students viewing and attempting them was relatively much larger.



**Fig. 1.8** Problem views without attempts

## 1.6 A mathematical model for MOOC perseverance

We were interested in whether the use of Mathlet-based problems held the attention of the less committed group of enrolled students better than problems not based on them. Disappointingly, we could find no statistically significant effect of Mathlet-based problems in this regard. But in studying this question a model of the population of online learners suggested itself.

We plotted the number of learners who attempted at most a fraction  $x$  of the problems against the fraction  $x$ . All the graphs, shown in Figure 1.9, showed a similar S-shaped curve. This pattern is dramatically bimodal, as would be clear also if we had plotted the number of learners doing exactly  $n$  problems against the number  $n$ . We choose this graphic for several reasons; for one thing, it shows the absolute number of students of various levels of commitment.

These graphs also suggest the following very simple model for MOOC the learner population, one that resonates with anyone who has taught a large university class.

There are two subpopulations of learners, which we dub “perfectionists” and “questers.”

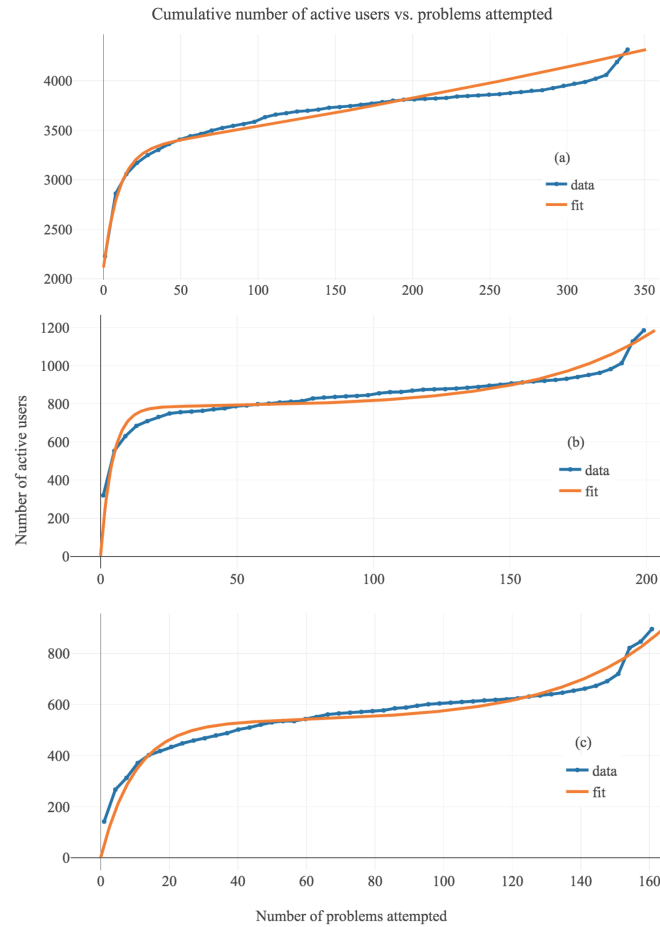
Each quester behaves probabilistically in the following way. The probability that she will try the fraction  $x$  or more of the problems declines exponentially with  $x$ : it is given by  $e^{-bx}$  for some positive constant  $b$ . This means that the probability that she tries  $x$  or fewer problems is  $1 - e^{-bx}$ .

In contrast, the perfectionist would like to do all the problems, but only approaches this ideal exponentially: The probability that a perfectionist fails to try the fraction  $y$  or more of the problems is  $e^{-ay}$  for some positive constant  $a$ . So the probability that he tries  $x$  or fewer is  $e^{-a(1-x)}$ .

In this scenario, if there are  $Q$  questers and  $P$  perfectionists in the population, then the expected number of learners trying the fraction  $x$  or fewer of the problems is given by

$$Q(1 - e^{-bx}) + Pe^{-a(1-x)}.$$

Of course, there is the additional side condition that  $P + Q$  is the total number of active students.



**Fig. 1.9** Cumulative problem attempts in: (a) 18.031, (b) 18.03L, (c) 18.032

We have done a least squares best fit analyses of this model to the data. The results are graphed in Figure 1.9. The model does not fit the data as well as we would like. We think that the reason for this is that the model of “questers” is inadequate. The persistence of this class of learner (measured in the number of problems they solve) appears not to decrease exponentially, but rather more slowly. This is a complex population of students, including eager students who joined late as well as desultory students who pick and choose the problems they view or attempt over the course of the module. This appears especially true in the first module, 18.031, which had a relatively large enrollment, exhibiting a diversity probably not found in the later modules. It was also twice as long as the other modules, and introductory in character, enabling “perfectionists” to enter late and further muddy the results. Refining this model presents an attractive research project.



## 1.7 Conclusion

This project began with the conviction that the attractive and responsive visualizations provided by the MIT Mathlets, combined with the interactivity provided by the Forum, would enhance student persistence in an online Differential Equations course. Sadly, we could not discern such an effect.

On the other hand, the occurrence of the Mathlets in problems might be thought to decrease student participation. We did not see that effect either: so at least they appear to do no harm.

We should point out that we have studied only participation, as measured by views or attempts at problems. We have nothing to say about the effect of the Mathlets on learning gains. We also have restricted our study to the MOOC version of these courses. Much of the same material is used today in the MIT residential courses, and the impact of the Mathlets on student learning and persistence would be a worthy topic of further study. It would be interesting for example to see how well binary model of the learner population sketched in Section 1.6 accounts for persistence in residential setting.

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