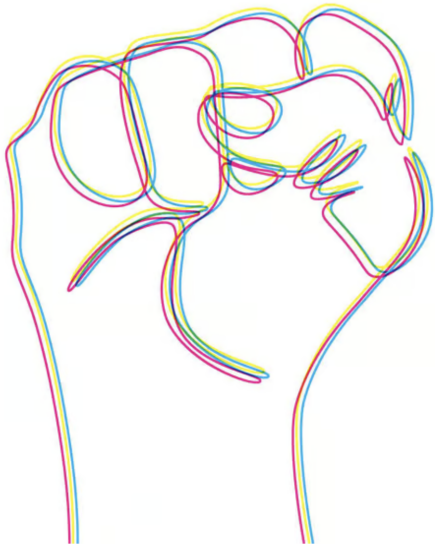
Theories and Realities

Haynes Miller
MIT

A celebration of the contributions of
Assaf Kfoury

Boston University, June 2026

Assaf as activist



S4P Conference: January, 2020

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Local organizing committee:

Assaf Kfoury

Haynes Miller

Raid Suleiman

Franz Ulm

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Imprisonment of Ubai Aboudi: Nov 13, 2019 – Oct 22, 2020

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The administrative detention of Ubai Aboudi

The State Department should call for his immediate release

By Assaf Kfoury, Haynes Miller, Nasser Rabbat, Raid Suleiman, and Franz-Josef Ulm
Updated December 11, 2019, 1:36 p.m.

A Palestinian man passes a closed shop in the West Bank city of Hebron, Monday. MAJDI MOHAMMED/ASSOCIATED PRESS

Bisan Center designation (Oct 22, 2021) and response

Israel outlaws Palestinian human rights groups, causing backlash from Israeli, international rights organizations

World Oct 22, 2021 4:01 PM EDT

JERUSALEM (AP) — Israel on Friday effectively outlawed six prominent Palestinian human rights groups by declaring them terrorist organizations, a major escalation of its decades-long crackdown on political activism in the occupied territories.

The declaration appeared to pave the way for Israel to raid their offices, seize assets, arrest staff and criminalize any public expressions of support for the groups. Most of the targeted organizations document alleged human rights violations by Israel as well as the Palestinian Authority, both of which routinely detain Palestinian activists.

Israeli and international rights groups condemned the move as an assault on civil society and expressed solidarity with the targeted organizations. Many noted that Israel already outlaws even peaceful political activities in the occupied West Bank. Palestinians want the territory — which Israel captured in the 1967 war — to form the main part of their future state.

The designated groups are Al-Haq, a human rights group founded in 1979, as well as the Addameer rights group, Defense for Children International-Palestine, the Bisan Center for Research and Development, the Union of Palestinian Women's Committees and the Union of Agricultural Work Committees.

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Detained Since: November 14, 2018.

Charges: Joining and financing a terrorist group (Article 13 of Egypt's Anti-Terrorism Law No. 94 of 2015)

Sentence: None.

Biography: Seif Fateen is a prominent chemical and environmental engineer based in Egypt where he served as assistant to the minister of higher education under the democratically elected government of Mohamed Morsi from 2012 to 2013. Seif was arrested without a warrant on November 14, 2018 when several members of the Egyptian National Security Agency first raided his home. They then proceeded to blindfold and transport him to an unknown detention facility, where he remained forcibly disappeared for more than nine months.

(Statement from the Congressional Tom Lantos Commission on Human Rights)

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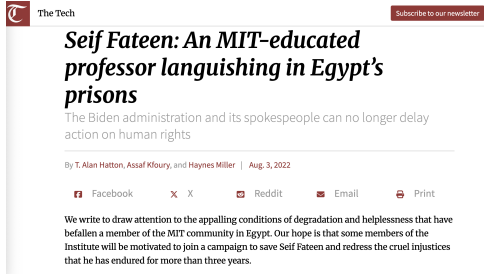

The Tech

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Seif Fateen: An MIT-educated professor languishing in Egypt's prisons

The Biden administration and its spokespeople can no longer delay action on human rights

By T. Alan Hatton, Assaf Kfoury, and Haynes Miller | Aug. 3, 2022

Facebook X Reddit Email Print

We write to draw attention to the appalling conditions of degradation and helplessness that have befallen a member of the MIT community in Egypt. Our hope is that some members of the Institute will be motivated to join a campaign to save Seif Fateen and redress the cruel injustices that he has endured for more than three years.

Assaf as mathematician

Assaf's MIT CS contacts

March 2020: Assaf invited me to look at the work of MIT CS graduate student Mirai Ikebuchi.

Term rewriting systems: What are they?

Quillen cohomology of Lawvere theories: Sounds interesting!

Applications of Homological Algebra to Equational Theories

by

Mirai Ikebuchi

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2021

Universal algebra

This perspective was introduced by William Lawvere in his 1963 thesis under Samuel Eilenberg at Columbia. An "algebraic category" can be described in terms of a small category T admitting finite products: a *cartesian category*.
The algebras for T ("*models*") are then functors to **Set** sending product diagrams to product diagrams.

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Example. FAb is the category of free abelian groups generated by the finite sets $\mathbf{n} = \{1, \ldots, n\}$, $n \geq 0$, and homomorphisms. FAb has finite coproducts, so its opposite category has finite products. An abelian group A determines a product-preserving functor: $FAb^{op} \to \mathbf{Set}$ sending $\mathbf{n}$ to $\mathrm{Hom}(\mathbb{Z}\mathbf{n}, A) = A^n$, and this determines an equivalence of categories

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Sorting. A "sorting" of a cartesian category is a choice of product diagram for every finite sequence of objects: a *rigid cartesian category* or *Lawvere theory*.

"Homology"

Dual(ing) perspectives on "homology":

(1) *(Algebraic)* The "homology" of a topological space is in a certain sense its *abelianization*. Some may be convinced by the example of a discrete space, whose homology is indeed the free abelian group it generates.

(2) *(Geometric)* If the space is simply connected, its homology tells you what building blocks to buy if you want to build the space. A "building block" here is a ball,

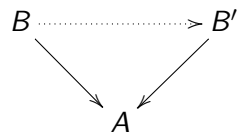
$$D^n = \{v \in \mathbb{R}^n : |v| \leq 1\}$$

The space gets built dimension-by-dimension, by gluing n-disks along their boundary spheres on to what you have already made.

Quillen homology

In *Homotopical Algebra*, 1967, Daniel Quillen provided a definition of what is to be meant by "homology" and "cohomology" in a very general algebraic setting $\mathbf{V}$, such as the category of models for a theory. Two ingredients:

Slice category $\mathbf{V}/A$: Objects (B, p), $B \in \mathbf{V}$, $p : B \downarrow A$. Morphisms:



A **Beck A-module** is an abelian object in $\mathbf{V}/A$: an object $B \downarrow A$ together with a section $A \uparrow B$ and a "multiplication" $B \times_A B \to B$ in $\mathbf{V}/A$ satisfying the axioms for an abelian group.

Beck modules

$\mathrm{Ab}(\mathbf{V}/A)$ is the category of "Beck A-modules": $\mathbf{Mod}_A$. (Jonathan Beck was another student of Sammy Eilenberg!) There is an adjoint pair

$$Ab_A : \mathbf{V}/A \rightleftarrows \mathrm{Ab}(\mathbf{V}/A) = \mathbf{Mod}_A : \text{forget}$$

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With $\mathbf{V} =$ associative K-algebras, $\mathbf{Mod}_A$ is the category of A-bimodules, and $\mathrm{Ab}_A(A \downarrow A) = \ker(A \otimes_K A \to A)$.

With $\mathbf{V} =$ groups, $\mathbf{Mod}_G$ is the category of $\mathbb{Z}G$-modules, and $\mathrm{Ab}_G(G \downarrow G) = \ker(\epsilon : \mathbb{Z}G \to \mathbb{Z})$.

With $\mathbf{V} =$ commutative K-algebras, $\mathbf{Mod}_A$ is the category of left A-modules, and $\mathrm{Ab}_A(A \downarrow A) = \Omega_{A/K}$.

Quillen homology

Definition. $HQ_*(A)$ is the sequence of derived functors Ab_A evaluated at $A \downarrow A$.

One can afflict Quillen homology with coefficients: $HQ_*(A; M)$.

Quillen homology describes how the algebra is built up from elementary ("free") pieces. The resolution involved in computing the derived functors expresses this procedure.

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With $\mathbf{V}$ = associative K-algebras, K a field: $HQ_n(A) = 0$ for $n > 0$, and $HQ_n(A; M) = HH_{n+1}(A; M)$, Hochschild homology.

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With $\mathbf{V}$ = commutative K-algebras: $HQ_*(A)$ is "André-Quillen homology" and rarely vanishes in positive dimension. Nevertheless quite well understood in many cases (Harrison, Avramov, Goerss, ...).

Homology of Lawvere theories

Example: "Term rewriting systems" provide a way of defining a Lawvere theory T in terms of a collection of operations obeying fixed relations.

Philippe Malbos and Samuel Mimram (following Jibladze and Pirashvili) set out to show how Quillen homology provides lower bounds on the complexity of a TRS presenting T.

Their pathfinding work was corrected and extended by Mirai Ikebuchi in her thesis. She defined the "degree" of a TRS and showed that for TRS's of degree d (assuming confluence and termination):

$\mathrm{rank} HQ_0(T; \mathbb{Z}/d)$ gives a lower bound on the number of operations;
$\mathrm{rank} HQ_1(T; \mathbb{Z}/d)$ gives a lower bound on the number of relations.

A step back: Categories

The Quillen homology of categories with fixed object set and functors fixing objects has been studied by Baues and Wirsching, Dwyer and Kan, and others.

Beck C-modules are "natural systems," that is,

$$\mathbf{Mod}_C = \mathbf{Fun}(\mathrm{Fac}_C, \mathbf{Ab})$$

where Fac_C is the "factorization category" or "twisted arrow category" of C: objects are morphisms in C, and given $f : X' \to X$ and $g : Y' \to Y$,

$$\mathrm{Fac}_C(f, g) = \left\{ \begin{array}{ccc} X' & \cdots\cdots\prec & Y' \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle g} \\ X & \cdots\cdots\succ & Y \end{array} \right\}$$

Category grading

"I-graded abelian group" means a choice of abelian group A_i for each $i \in I$.

The category structure of C determines a (non-symmetric) monoidal structure on the category of $\mathrm{Mor}(C)$-graded abelian groups:

$$(A_* \otimes B_*)_h = \bigoplus_{f,g \text{ s.t. } fg=h} A_f \otimes B_g$$

The initial monoid with respect to this monoidal structure has $(\widetilde{\mathbb{Z}}C)_f = \mathbb{Z}\iota_f$, with

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Observation: $\mathbf{Mod}_C$ is equivalent to the category of $\widetilde{\mathbb{Z}}C$-bimodules.

Under this identification, the cochain complex used by Hans Baues and Günther Wirsching their "cohomology of small a category with coefficients in a natural system" is simply the Hochschild complex computing the Hochschild cohomology $HH^*(\widetilde{\mathbb{Z}}C; M)$.

Quillen cohomology of categories

Let A_* be C-graded unital associative K-algebra. There is a canonical "Hochschild complex"

$$A_* \leftarrow A_* \otimes A_* \Leftarrow A_* \otimes A_* \otimes A_* \Leftarrow\!\Leftarrow \cdots$$

that is split exact as left or right A_*-modules. It expresses the "noncommutative Kähler differentials" as a kernel:

$$0 \leftarrow A_* \leftarrow A_* \otimes A_* \leftarrow \mathrm{Ab}_{A_*}(A_* \downarrow A_*) \leftarrow 0$$

Following Quillen, this quickly implies

Theorem

Suppose A_f is K-flat for all $f \in \mathrm{Mor}(C)$. Then $HQ_n(A_*) = 0$ for all $n > 0$.

Corollary (maybe due to Dwyer and Kan)

Quillen and Baues-Wirsching cohomology for small categories coincide.

Happy birthday, Assaf!