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SOME REMARKS ON Tors Ω_{Sp}^*.

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The Ω_{Sp}^*-theory of cobordisms appeared in [4] and was developed in [11],[8],[3] and other works.

A series of free generators was constructed by Stong [14].

Great difficulty was encountered in treating the question on the 2-primary torsion.

Ray [12] constructed a series of multiplicatively indecomposable elements of the second order.

By analogy with [10] here we construct a series of finite order elements and relations in the Ω_{Sp}^*-theory.

1. Let

$$e(\theta\otimes\zeta)=x+y+\sum\varphi_k y^k,$$

where θ is a canonical complex linear fibering over $RP(\infty)$, ζ a canonical simplectic fibering over $HP(\infty)$, e - Euler's class in the Ω_{Sp}^*-theory, $x=e(2\theta)$, $y=e(\zeta)$ and $\varphi_k\in\Omega_{Sp}^{4-4k}RP(\infty)$.

Remark 1. Let $\Psi: RP(1)\to RP(\infty)$ be a canonical imbedding. Then

$$\Psi^*\varphi_k=\theta_k\cdot s$$

where $\theta_k\in\Omega_{Sp}^{3-4k}$ are Ray's elements [12] and s is the generator $\Omega_{Sp}^{1}RP(1)$.

Remark 2. It follows from [13],[2],[10] that for $k>0$, classes φ_{2k+1} depend functionally on classes $\varphi_1, \varphi_2, \varphi_4, \ldots, \varphi_{2k}$;

while classes $\varphi_1, \varphi_2, \varphi_4, \ldots, \varphi_{2k}$ are functionally independent.

By virtue of the property of two-valued formal groups and Conner-Floyd classes in the Ω^*_{Sp}-theory [1] we have

Lemma 1. $4e(\theta\otimes\zeta) = 2\Theta_1(x,y)$,

$$6e^2(\theta\otimes\zeta) = \Theta_1^2(x,y) + 2\Theta_2(x,y) = \sum \mathfrak{a}_{ij} x^i y^j$$

where $\mathfrak{a}_{ij} \in \Omega^*_{Sp}$,

$$4e^3(\theta\otimes\zeta) = 2\Theta_1(x,y)\cdot\Theta_2(x,y),$$

$$e^4(\theta\otimes\zeta) = \Theta_2^2(x,y),$$

where $\Theta_1(x,y)$ and $\Theta_2(x,y)$ are coefficients of the two-valued formal group.

Remark 3. In the Ω^*_{SC}-theory of cobordisms

$$2e(\theta\otimes\zeta) = \Theta_1(x,y),$$

$$e^2(\theta\otimes\zeta) = \Theta_2(x,y).$$

Corollary 1.

$$6\varphi_{2k}^2 = -12\varphi_{4k}x - 12\sum_{i+j=4k}\varphi_i\varphi_j + \sum_{i=1}^{\infty}\mathfrak{a}_{i,4k}x^i,$$

$$6\varphi_1^2 = -6 - 12\varphi_1 - 12x\varphi_2 + \sum_{i=0}^{\infty}\mathfrak{a}_{i,2}x^i.$$

Definition. $R(\omega) =$

$$= \langle \varphi_1^{k_0} \varphi_{2i_1}^{k_1} \cdots \varphi_{2i_n}^{k_n}, RP(4m+3) \rangle = \varepsilon D_{Sp} \varphi_1^{k_0} \varphi_2^{k_1} \cdots \varphi_{2i_n}^{k_n} \big| RP(4m+3),$$

where $k_0 \geq 0$, $k_i > 0$ are integers, D_{Sp} the duality operator of Atiyah-Poincaré[7], ε the argumehtation,

$$1 \leq i_1 < i_2 < \cdots < i_n;$$

$$\omega = \begin{pmatrix} k_0 & k_1 & & k_n & \\ 1, & 2i_1, & \cdots & 2i_n, & 4m+3 \end{pmatrix}.$$

Remark 4. By virtue of Remark 3 we consider in the Ω^*_{SC} - theory the following elements $R(\omega)$ [10]:

$$\omega = \begin{pmatrix} k & 1 & 1 & & 1 & \\ 1, & 2i_1, & 2i_2, & \cdots & 2i_n, & 4m+1 \end{pmatrix} \quad \text{where} \quad k = 0, 1.$$

Remark 5. From the formula $e^4(\theta \otimes \zeta) = \theta_2^2(x, y)$ it follows that φ_{2i}^4 are decompasable elements in the ring $\Omega^*_{Sp} RP(\infty)$. Therefore in what follows we shall always consider the case $k_i \leq 3$.

Theorem 1. If $k_i > 1$ for some $i \geq 0$, then

$$2R(\omega) = K(i, \omega),$$

where $K(i,\omega)$ are decomposable elements in the ring Ω^*_{Sp}.

Remark 6. To constructs elements $K(i,\omega)$ we need the definition of $R(\omega)$, Corollary 1 and the formula

$$D_{Sp}(x) \big| RP(4m+3) = [RP(4m-1)]$$

in the Ω^*_{Sp} -theory.

Remark 7. The elements $R(\omega)$ and $K(i,\omega)$ are defined correctly and Theorem 1 is the analogue of the Massey product in the Ω^*_{Sp} -theory. The Massey product in the cobordism theory has hitherto been the only means for constructing new elements in the Ω^{*}_{Sp} -theory [6],[8],[3].

Corollary 2. The element K_{111} in the dimension 111

1. $K_{111} = R\left(\frac{2}{2}, \frac{1}{8}, \frac{1}{16}, 15\right) = \langle \varphi_2^2 \varphi_8' \varphi_{16}, RP(15) \rangle$

coincides with Kochman's element [8].

2. $4K_{111} = 0$.

Hypothesis: 1. Among $K(i,\omega)$ there exist non-trivial elements.

2. $2K_{111} \neq 0$

Remark 8. Kochman [8], Ray, Ivanovski, Vershinin, Botvinik put forward a hypothesis that fourth order appear for the first time in dimension 111.

2. Definition. A non-trivial element on which all Landveber-Novikov operations [5],[9] act trivially is called the primitive element in the ring .

Examples of primitive elements:

1. $\theta_1 \in \Omega^{1}_{Sp}$,
2. $\theta_1^2 \in \Omega^{2}_{Sp}$,
3. $\theta_2 \theta_4^2 \in \Omega^{31}_{Sp}$.

Theorem 2. For any primitive element ε in the ring Ω^{*}_{Sp} there exists an element $\delta \in \Omega^{*}_{Sp}$ such that

$$4\varepsilon = 8\delta.$$

Remark 9. The theorem is proved using Steenrod-Dick operations, an exactsequence of the covering in the Ω^{*}_{Sp} -theory, transfer properties for the covering and elements of the two-valued formal group theory.

Definition. OM is a set of maximum order elements in Ω^{n}_{Sp}, where n is the minimum dimension in which there exist elements of order higher than 4.

Corollary 3. If in OM there exists a primitive element, then

$$4\,\mathrm{Tors}\,\Omega^*_{Sp} = 0.$$

Hypothesis. 1. OM contains a primitive element

2. $2\,\mathrm{Tors}\,\Omega^*_{Sp} \subset \theta_1 \Omega^*_{Sp}$.

3. $2\,\mathrm{Im}(\mathrm{Tors}\,\Omega^*_{Sp} \to \Omega^*_{SC}) = 0$.

Remark 10. As reported by B.I. Botvinik, $16\,\mathrm{Tors}\,\Omega^*_{Sp} = 0$.

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