

## Problem set 7, due November 21

The following exercises are taken from the book, references are from Ross, Edition 9, chapter 7. Due Friday November 21 at the beginning of lecture.

**Problem 1 (Problem 29)** Let  $X_1, \dots, X_n$  be independent and identically distributed random variables. Find

$$\mathbb{E}[X_1 | X_1 + \dots + X_n = x]$$

**Problem 2 (Problem 67)** Consider a gambler who, at each gamble, either wins or loses his bet with respective probabilities  $p$  and  $1-p$ . A popular gambling system known as the Kelly strategy is to always bet the fraction  $2p-1$  of your current fortune if  $p > 1/2$ . Compute the expected fortune after  $n$  gambles of a gambler who starts with  $x$  units and employs the Kelly strategy.

**Problem 3 (Problem 50)** The joint density of  $X$  and  $Y$  is given by

$$f(x, y) = \frac{1}{y} e^{-x/y-y}, \quad 0 < x < \infty, \quad 0 < y < \infty.$$

Compute  $E[X^2 | Y = y]$

**Problem 4 (Problem 53)** A prisoner is trapped in a cell containing 3 doors. The first door leads to a tunnel that returns him to his cell after two days travel. The second leads to a tunnel that returns him to his cell after four days travel. The third door leads to freedom after one day travel. It is assumed that the prisoner will always select door 1, 2 or 3 with respective probability 0.5, 0.3 and 0.2, what is the expected number of days until the prisoner reaches freedom ?

**Problem 5 (Theoretical exercise 21)** Let  $X_{(i)}$  denote the order statistics of  $n$  independent uniform on  $[0, 1]$ .

(1) Compute  $Var(X_{(i)})$

(2) For which value of  $i$  is the variance minimum ? For which value of  $i$  is the variance maximum?

**Problem 6 (Theoretical exercise 24)** Prove the Cauchy-Schwartz inequality, namely:

$$(\mathbb{E}[XY])^2 \leq \mathbb{E}[X^2]\mathbb{E}[Y^2]$$

**Problem 7 (Theoretical exercise 33)** A coin that lands on heads with probability  $p$  is continually flipped. Compute the expected number of flips that are made until a string of  $r$  heads in a row is obtained. *Hint:* Condition on the time of the first occurrence of tails to obtain the equation

$$\mathbb{E}[X] = (1-p) \left( \sum_{i=1}^r p^{i-1} (i + \mathbb{E}[X]) + \sum_{i=r+1}^{\infty} p^{i-1} r \right).$$