Problem set 4, due october 3

The following exercises are taken from the book, references are from Ross, Edition 9, chapter 4. Due Friday October 3 at the beginning of lecture.

Problem 1 Show that for a nonnegative integer valued random variable N

$$\mathbb{E}[N] = \sum_{i \geq 1} P(N \geq i).$$

Show that

$$\mathbb{E}[N^2] - \mathbb{E}[N] = 2 \sum_{i=0}^{\infty} i P(N > i).$$

Problem 2 (Problem 1) Two balls are chosen randomly from an urn containing 8 white, 4 black and 2 orange balls. Suppose that we win 2 dollars for each black selected and lose one for each white selected. Let X denote our winnings. What are the possible values of X and what are the probabilities associated with each value?

Problem 3 (Problem 50) Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are

(1) h, t, t (meaning that the first flip results in heads, the second in tails, and the third in tails);
(2) t, h, t

Problem 4 Consider an infinite sequence of independent tosses of a coin that comes up heads with probability p.

(1) Let X be the first toss when a head appears. Compute the probability distribution of X, its expectation and variance.
(2) Let Y be the third time heads appears. Compute the expectation and variance of Y.

Problem 5 (Problem 20) A gambling book recommends the following "winning strategy" for the game of roulette: Bet 1 dollar on red (either you lose your dollar or win a dollar). If red appears (with probability 18/38) then take the 1 dollar profit and quit. If red does not appear and you lose this bet, make 1 dollar addition; bet on red on the next two spins and then quit. Let X denote your winnings when you quit.

(1) Find $P(X > 0)$
(2) Are you convinced that the strategy is indeed winning? (Explain)
(3) Find $E[X]$

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Exercise 6 (Theoretical exercise 13) Let X be a binomial random variable with parameters (n, p). What value of p maximizes $P(X = k), k = 0, 1, \ldots n$? This is an example of statistical method used to estimate p when a binomial (n, p) random variables is observed to equal k. If we assume that n is known, then we estimate p by choosing that value of p that maximizes $P(X = k)$. This is known as the method of maximum likelihood estimation. .