

Problem set 1, due september 12

Most exercises are taken from the book, references are from Ross, Edition 9, chapter one.

Due Friday september 12 at the beginning of the lecture (stapled, with your name clearly written on the first page)

- **Problem 1 (Problem 24)** Expand $(3x^2 + y)^5$

- **Problem 2 (Extended version of problem 10)** In how many ways can 8 people be seated in a row if

- a) There are no restrictions on the seating arrangement?
 - b) Persons A and B must sit next to each other?
 - c) There are 4 men and 4 women and no 2 men or 2 women can sit next to each other ?
 - d) there are 4 married couples and each couple must sit together ?
 - e) There are 3 men and 5 women and the women must seat together and the men together
- f) What can you say about these questions if they are seated around a round table ?

- **Problem 3 (Problem 28)** If 8 new teachers are to be divided among 4 schools, how many divisions are possible ? What if each school must receive 2 teachers ?

- **Exercise 4 (Theoretical exercise 12)** Consider the following combinatorial identity:

$$\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$$

- Prove this identity by a combinatorial argument t for the above by considering a set of n people and determining, in two ways, the number of possible selections of a committee of any size and a chairperson for the committee. Hint:
 - How many possible selections are there of a committee of size k and its chairperson?
 - How many possible selections are there of a chairperson and the other committee members.
- Optional: Prove this identity analytically by using the binomial theorem and differentiation.

- **Exercise 5 (Theoretical exercises 8 and 9)** Prove that for n, m, r integer numbers

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$$\binom{n+m}{r} = \binom{n}{0} \binom{m}{r} + \binom{n}{1} \binom{m}{r-1} + \cdots + \binom{n}{r} \binom{m}{0}$$

Hint: Consider a group of n men and m women. How many group size of size r are possible ?

- Deduce from the previous that

$$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2$$

- **Exercise 6** In France, license plates have 4 letters and 3 digits. How many license plates if there is no repetition of letters and digits ? what if we must have 2 letters, then 3 digits, and then 2 consecutive letters (and they can be equal)?

- **Exercise 7** Suppose you have 20 (distinguishable) books and 4 (distinguishable) people. How many ways to divide the 20 books among the 4 people with each person getting exactly 5 books ?