Solution Page Linear Constant Coefficient Equations

First order $\dfrac{dy}{dt} = ay + f(t)$ **Second order** $A\dfrac{d^2y}{dt^2} + B\dfrac{dy}{dt} + Cy = f(t)$

Nth order $A_N\dfrac{d^Ny}{dt^N} + \cdots + A_1\dfrac{dy}{dt} + A_0 y = (A_N D^N + \cdots + A_0)y = P(D)y = f(t)$

Null solutions y_n have $f(t) = 0$ Substitute $y = e^{st}$ to find the N exponents s

First order	$\dfrac{d}{dt}(e^{st}) = ae^{st}$	$s = a$ and $y_n = ce^{at}$
Second order	$As^2 + Bs + C = 0$	$y_n = c_1 e^{s_1 t} + c_2 e^{s_2 t}$
Nth order	$P(s) = 0$	$y_n = c_1 e^{s_1 t} + \cdots + c_N e^{s_N t}$

Exponential response to $f(t) = e^{ct}$ Step response for $c = 0$ Look for $y = Ye^{ct}$

First order	$\dfrac{d}{dt}\left(Ye^{ct}\right) - aYe^{ct} = e^{ct}$	$y_p = \dfrac{e^{ct}}{c-a}$ has $Y = \dfrac{1}{c-a}$
Second order	$Y(Ac^2 + Bc + C)e^{ct} = e^{ct}$	$y_p = \dfrac{e^{ct}}{Ac^2 + Bc + C} = Ye^{ct}$
Nth order	$YP(c)e^{ct} = e^{ct}$	$y_p = \dfrac{e^{ct}}{P(c)}$ or $\dfrac{te^{ct}}{P'(c)}$ when $P(c) = 0$

Fundamental solution $g(t)$ = Impulse response when $f(t) = \delta(t)$

First order	$g(t) = e^{at}$	starting from $g(0) = 1$
Second order	$g(t) = \dfrac{e^{s_1 t} - e^{s_2 t}}{A(s_1 - s_2)}$	starting from $g(0) = 0$ and $g'(0) = 1/A$
Undamped	$g(t) = \dfrac{\sin \omega_n t}{A\omega_n}$	underdamped $g(t) = e^{-Z\omega_n t}\dfrac{\sin \omega_d t}{A\omega_d}$
Nth order	$g(t) = y_n(t)$	$g(0) = g'(0) = \ldots = 0, g^{(N-1)}(0) = 1/A_N$

Very particular solution for each driving function $f(t)$: zero initial conditions on y_{vp}

Multiply input at every time s
by the growth factor over $t - s$

$$y(t) = \int_0^t g(t-s)\, f(s)\, ds$$

Undetermined coefficients	Direct solution for special $f(t)$ in Section 2.6
Variation of parameters	$y_p(t)$ comes from $y_n(t)$ in Section 2.6
Solution by Laplace transform	Transfer function = transform of $g(t)$ in Section 2.7
Solution by convolution	$y(t) = g(t) * f(t)$ in Section 8.6