

### (Problem 2.3-3)

The following lines of code create the matrices, compute the six different least squares approximations, and show the result in a way that makes it easy to compare each of the coefficients.

```
V=flipplr(vander((0:49)/49));
A=V(:,1:12);
b=cos(0:.08:3.92)';

xa=(A'*A)\(A'*b);
[Q,R]=clogs(A); xb=R\ (Q'*b);
[Q,R]=mgs(A); xc=R\ (Q'*b);
[W,R]=house(A); Q=formQ(W); xd=R\ (Q(:,1:12) '*b);
xe=A\b;
[Q,R]=qr(A,0); xf=R\ (Q'*b);

format long
coeffs=[xa,xb,xc,xd,xe,xf]'
```

The printout is

Columns 1 through 4

<u>1.00000000933257</u>	<u>-0.00000290567458</u>	<u>-7.68308776485945</u>	<u>-0.00169530287455</u>
<u>0.99999920491753</u>	<u>0.00018008412375</u>	<u>-7.68892310403998</u>	<u>0.07097733960275</u>
<u>0.9999999790382</u>	<u>0.00000051294498</u>	<u>-7.68321706569439</u>	<u>0.00022083245185</u>
<u>1.00000000070510</u>	<u>-0.00000029643270</u>	<u>-7.68318691743746</u>	<u>-0.00022062970771</u>
<u>1.00000000070511</u>	<u>-0.00000029643293</u>	<u>-7.68318691743328</u>	<u>-0.00022062974385</u>
<u>1.00000000070511</u>	<u>-0.00000029643272</u>	<u>-7.68318691743935</u>	<u>-0.00022062967276</u>

Columns 5 through 8

<u>9.85186632950063</u>	<u>-0.06102468492730</u>	<u>-4.86452221958406</u>	<u>-0.31830540225001</u>
<u>9.38830444630009</u>	<u>1.64581702834279</u>	<u>-8.69605853647215</u>	<u>4.99166341407025</u>
<u>9.83712505954158</u>	<u>0.00578476580229</u>	<u>-5.05465774349718</u>	<u>0.03106251054057</u>
<u>9.84048754969992</u>	<u>-0.00932461159254</u>	<u>-5.01197587819606</u>	<u>-0.04685413668128</u>
<u>9.84048754987227</u>	<u>-0.00932461207708</u>	<u>-5.01197587739097</u>	<u>-0.04685413739574</u>
<u>9.84048754942819</u>	<u>-0.00932461041494</u>	<u>-5.01197588133249</u>	<u>-0.04685413134327</u>

Columns 9 through 12

<u>1.73999932805678</u>	<u>-0.21529974828028</u>	<u>-0.21387234372243</u>	<u>0.05391198296639</u>
<u>-2.66555474599036</u>	<u>1.75133778152025</u>	<u>-0.53926578103067</u>	<u>0.02948908485208</u>
<u>1.32597309062972</u>	<u>0.09028403114867</u>	<u>-0.34162518024562</u>	<u>0.07701647471226</u>
<u>1.41777858474053</u>	<u>0.02287193901610</u>	<u>-0.31357240519150</u>	<u>0.07196408562391</u>
<u>1.41777858489809</u>	<u>0.02287193930069</u>	<u>-0.31357240544640</u>	<u>0.07196408569042</u>
<u>1.41777857891566</u>	<u>0.02287194295893</u>	<u>-0.31357240669823</u>	<u>0.07196408587221</u>

The wrong digits are marked with underlines. The normal equations are very inaccurate, the coefficient with the largest relative error is more than a magnitude too large, indicating that the normal equations exhibit instability. The condition number of  $A^*A$  is  $1.42 \cdot 10^{16}$ , and it is indeed hard to get good accuracy with 16 digits precision. The solution with the classical Gram-Schmidt algorithm is even worse, and the modified Gram-Schmidt algorithm is also very bad. The last three methods produce results with essentially the same accuracy, although the solutions differ in several of the last digits since the problem is ill-conditioned.