18.085 **Quiz 3** **May 13, 1996**

Your name is ______________________ Grading:

Open book exam. 1.

2.

Answer Question 1 below and on the next two blank pages. 3.

Total:

1. In this problem $F(x) = \dfrac{1}{2a}$ for x between $-a$ and a.

   (a) Find the Fourier series coefficients a_k and b_k and c_k if $f(x) = 0$ for $|a| < x \leq \pi$.

   (b) What limit does the coefficient c_k approach as $a \to 0$? What function does $f(x)$ approach as $a \to 0$?

   (c) Parseval's formula gives what numerical value for which infinite series of squares?

   (d) In general, if $f(x) = \sum c_k e^{ikx}$ and $g(x) = \sum d_n e^{inx}$, find a formula for $\int_{-\pi}^{\pi} f(x)\overline{g(x)}\,dx$.

   (e) What equality do you get from part (d) for the particular $f(x)$ in this problem (and any $g(x)$)? What is the limit of your equality as a approaches zero?

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2. (a) State the orthogonality equation(s) that make the *discrete* Fourier transform a big success.

(b) What is the ordinary convolution (not circular) of the vector $y = (1, 1, 1, 1)$ with itself? What is the circular convolution?

(c) This convolution $y * y$ corresponds to exactly what multiplication in the frequency domain? What are the c's?

3. This problem is about the differential equation

$$-u'' + 2u' + 3u = \delta(x) \qquad \text{for } -\infty < x < \infty\,.$$

(a) Solve this *separately* for $x > 0$. Then solve when $x < 0$. The two solutions include arbitrary constants but they must approach zero at infinity. What are the jump conditions at $x = 0$? What is the overall solution to the equation?

(b) Solve by Fourier transform to find $\widehat{u}(k)$. How many of these functions are continuous: $u(x)$, $u'(x)$, $u''(x)$?

(c) Suppose the right side of the equation is $f(x)$ instead of $\delta(x)$.

What is an integral formula for the solution now?

(d) Find the solution $u(x)$ if $f(x) = \dfrac{1}{2a}$ for $|x| \le a$ and $f(x)$ is *zero everywhere else.*