18.085 Quiz 1 3/11/96

Your name is ______________________ Grading: 1

2

3

4

Total

1.

(a) Write down the 6×5 incidence matrix A_0.

(b) Find *all* solutions to $A_0^T y = 0$. (Remember the loops.)

(c) Ground nodes 4 *and* 5: $x_4 = x_5 = 0$. Write out explicitly 9 equations for $x_1\, x_2\, x_3\, y_1 \cdots y_6$ assuming $\underline{c = 2}$ in each edge.

(d) Is the 9 by 9 matrix created in (c) positive definite? Why or why not? What is the definition of a positive definite matrix?

(e) Find the matrix $A^T C A$ for this network. How do you know it is positive definite?

2. To the top joint of a tree house,
   my kids pinned a 45° flagpole:

   (a) Find the matrix A for this truss.

   (b) Find all mechanisms x that solve $Ax = 0$.

   How many are there? Draw a full set of independent mechanisms. Could the flagpole 5 have been better placed to give a completely stable truss?

   (c) Suppose joint 4 is also fixed. Is this new truss statically determinate or indeterminate or unstable?

   (d) With joint 4 fixed and all $c_i = 1$, what is A and what forces $f_H^1\ f_V^1\ f_H^2\ f_V^2\ f_H^3\ f_V^3$ will produce $x_H^2 = 1$ and all other displacements $= 0$?

3. Given m equations $Ax = b$ in n $(n < m)$ unknowns,

   (a) The least squares solution $\bar{x}$ is the vector that minimizes

   ____ .

   (b) This $\bar{x}$ actually solves $A\bar{x} = b$ when b is in

   ____ .

   (c) The equation $A^T y = 0$ says geometrically that e is perpendicular to

   ____ .

   (d) There is a unique solution for e (True or False) because

   ____ .

   (e) The equation for $\bar{x}$ is

   ____ .

   (f) This has a unique solution $\bar{x}$ (always? any condition on A?) if

   ____ .

4. Instead of a line of springs we have a building fixed at the *bottom.*

(a) Find the 3 equations that give the displacements x_1, x_2, x_3 in terms of the forces m_1g, m_2g, m_3g and the spring constants c_1, c_2, c_3 of the walls.

(b) The matrix in $K \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} m_1g \\ m_2g \\ m_3g \end{bmatrix}$ can be factored into $K = LU$. What are L and U? L has ones on the diagonal; c_1, c_2, c_3 are some positive numbers.

(c) What are x_1, x_2, x_3 if $c_1 = c_2 = c_3 = 1$?