Problem set 5

This problem set is due in class on Thursday May 11th, 2017.

1. Exercise 5-4 of the notes on matroids.

2. Exercise 5-5 of the notes on matroids.

3. Exercise 5-8 of the notes on matroids.

4. Suppose we are given a matroid $M = (E, \mathcal{I})$, a weight function $w : E \to \mathbb{R}$, and we are interested in finding a base $B$ of $M$ of maximum total weight $w(B)$. We could use the greedy algorithm, but suppose instead we use the following algorithm. Start from any base $B_1$ of $M$. At iteration $i$, if there exists $e \not\in B_i$ and $f \in B_i$ with $w(e) > w(f)$ such that $B_i + e - f$ is a base of the matroid then let $B_{i+1} = B_i + e - f$ and continue; otherwise (if there is not such $e$ and $f$) stop and output $B_i$. 

(a) Prove that this algorithm correctly outputs a base of maximum total weight.

(b) Suppose now that among all possible $e$ and $f$ satisfying the condition above, we should the one for which $w(e) - w(f)$ is maximum. Prove that $w(B_{i+1}) - w(B_i) \geq \frac{1}{r}(w(B^*) - w(B_i))$, where $r = r(E)$ is the size of any base, and $B^*$ is an optimum base. (This result can be used to bound the number of iterations of such an algorithm.)

5. Exercise 6-1 of the notes on matroid intersection.

6. Show the derivation of Theorem 6.2 from Theorem 6.1, from the notes on matroid intersection.