Massachusetts Institute of Technology
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18.453: Combinatorial Optimization
May 2nd, 2017

Problem set 5

This problem set is due in class on Thursday May 11th, 2017.

1. Exercise 5-4 of the notes on matroids.

2. Exercise 5-5 of the notes on matroids.

3. Exercise 5-8 of the notes on matroids.

4. Suppose we are given a matroid $M = (E, \mathcal{I})$, a weight function $w : E \to \mathbb{R}$, and we are interested in finding a base B of M of maximum total weight $w(B)$. We could use the greedy algorithm, but suppose instead we use the following algorithm. Start from any base B_1 of M. At iteration i, if there exists $e \notin B_i$ and $f \in B_i$ with $w(e) > w(f)$ such that $B_i + e - f$ is a base of the matroid then let $B_{i+1} = B_i + e - f$ and continue; otherwise (if there is not such e and f) stop and output B_i.

   (a) Prove that this algorithm correctly outputs a base of maximum total weight.

   (b) Suppose now that among all possible e and f satisfying the condition above, we should the one for which $w(e) - w(f)$ is maximum. Prove that $w(B_{i+1}) - w(B_i) \geq \frac{1}{r}(w(B^*) - w(B_i))$, where $r = r(E)$ is the size of any base, and B^* is an optimum base. (This result can be used to bound the number of iterations of such an algorithm.)

5. Exercise 6-1 of the notes on matroid intersection.

6. Show the derivation of Theorem 6.2 from Theorem 6.1, from the notes on matroid intersection.