SOLUTIONS FOR QUIZ 1

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Solve the integral*

$$\int \cos^5 t \sin^6 t \, dt.$$

Solution: Taking $u = \sin t$ one has that $du = \cos t \, dt$ and, therefore,

$$\begin{aligned}
\int \cos^5 t \sin^6 t \, dt &= \int (1 - \sin^2 t)^2 \sin^6 t \, \cos t \, dt \\
&= \int (1 - u^2)^2 u^6 \, du \\
&= \int u^6 - 2u^8 + u^{10} \, du \\
&= \frac{u^7}{7} - 2\frac{u^9}{9} + \frac{u^{11}}{11} + C \\
&= \frac{\sin^7 t}{7} - 2\frac{\sin^9 t}{9} + \frac{\sin^{11} t}{11} + C.
\end{aligned}$$

□

Problem 2. *Solve the integral*

$$\int x^2 5^x \, dx.$$

Solution: Let us use integration by parts taking $u = x^2$ and $dv = 5^x \, dx$. In this case, $du = 2xdx$ and $v = 5^x / \ln 5$, which implies that

$$\int x^2 5^x \, dx = \frac{x^2 5^x}{\ln 5} - \frac{2}{\ln 5} \int x 5^x \, dx. \tag{0.1}$$

To compute the integral in the right-hand side of (0.1), we applying again integration by parts setting this time $u = x$ and $dv = 5^x \, dx$. Then $du = dx$ and $v = 5^x / \ln 5$, and we obtain

$$\int x^2 5^x \, dx = \frac{x^2 5^x}{\ln 5} - \frac{2}{\ln 5}\left(\frac{x 5^x}{\ln 5} - \frac{1}{\ln x} \int 5^x \, dx\right) = \frac{x^2 5^x}{\ln 5} - \frac{2x 5^x}{(\ln 5)^2} + \frac{2 \cdot 5^x}{(\ln 5)^3} + C.$$

□

Problem 3. *Evaluate the integral*

$$\int_0^2 \frac{dt}{\sqrt{4+t^2}}.$$

Solution: Taking $t = 2\tan u$, we get $\sqrt{4+t^2} = 2\sec u$ and $dt = 2\sec^2 u\, du$. As result,

$$\int_0^2 \frac{dt}{\sqrt{4+t^2}} = \int_0^{\pi/4} \sec u\, du = \ln|\sec u + \tan u|\Big|_0^{\pi/4} = \ln(\sqrt{2}+1).$$

□

Problem 4. *Solve the integral*

$$\int \frac{x^3 - 3x^2 + 9x + 14}{x^2 - 4x + 13}\, dx.$$

Solution:

$$\begin{aligned}\int \frac{x^3 - 3x^2 + 9x + 14}{x^2 - 4x + 13}\, dx &= \int \left(x + 1 + \frac{1}{x^2 - 4x + 13}\right) dx \\ &= \frac{x^2}{2} + x + \int \frac{dx}{(x-2)^2 + 3^2} \\ &= \frac{x^2}{2} + x + \frac{1}{3}\tan^{-1}\left(\frac{x-2}{3}\right) + C.\end{aligned}$$

□

Problem 5. *Solve the integral*

$$\int \sec^4 x \tan^3 x\, dx.$$

Solution: Taking $u = \tan x$ we obtain $du = \sec^2 x\, dx$. Thus,

$$\begin{aligned}\int \sec^4 x \tan^3 x\, dx &= \int (1 + \tan^2 x)\tan^3 x\, \sec^2 x\, dx \\ &= \int (1+u^2)u^3\, du \\ &= \frac{u^4}{4} + \frac{u^6}{6} + C \\ &= \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C.\end{aligned}$$

□

Problem 6. *Solve the integral*

$$\int \frac{dx}{\sqrt{x^2+6x+10}}.$$

Solution: Taking $x+3=\tan u$, one finds that $dx=\sec^2 u\,du$. Therefore

$$\begin{aligned}\int \frac{dx}{\sqrt{x^2+6x+10}} &= \int \frac{dx}{\sqrt{(x+3)^2+1}}\\ &= \int \sec u\,du\\ &= \ln|\sec u+\tan u|+C\\ &= \ln|\sqrt{x^2+6x+10}+x+3|+C.\end{aligned}$$

□

Problem 7. *Solve the integral*

$$\int \frac{x^2-7}{x^2+4x+3}\,dx.$$

Solution:

$$\begin{aligned}\int \frac{x^2-7}{x^2+4x+3}\,dx &= \int \left(1-\frac{4x+10}{x^2+4x+3}\right)dx\\ &= x-\int \frac{3}{x+1}\,dx-\int \frac{1}{x+3}\,dx\\ &= x-3\ln|x+1|-\ln|x+3|+C.\end{aligned}$$

□

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.