PROBLEM SET 9: AREA OF A SURFACE OF REVOLUTION

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Find the total area of of the surface resulting from rotating the curve*

a) $y = x^3,\ 0 \le x \le 2$ *about the x-axis;*

b) $y = \cos\left(\frac{1}{2}x\right),\ 0 \le x \le \pi$ *about the x-axis;*

c) $y = \frac{x^3}{6} + \frac{1}{2x},\ 1/2 \le x \le 1$ *about the x-axis;*

d) $x^{2/3} + y^{2/3} = 1,\ 0 \le y \le 1$ *about the y-axis;*

e) $x = \frac{1}{4}y^2 - \frac{1}{2}\ln y,\ 1 \le y \le 2$ *about the y-axis.*

Problem 2. *Show that the total area of the surface of revolution obtained by rotating the curve* $y = 1/x$ *with* $x \ge 1$ *about the x-axis is infinite.*

Problem 3. *Show that the total area of the surface of revolution obtained by rotating the curve* $y = e^{-x}$ *with* $x \ge 0$ *about the x-axis is finite.*

Problem 4. *Show that the area of the sphere of radius* r *is* $4\pi r^2$.

Problem 5. *Find the total area of of the surface resulting from rotating the circle* $x^2 + y^2 = r^2$ *about the line* $y = r$.

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.