

PROBLEM SET 20: SOME APPLICATIONS OF TAYLOR SERIES

Note: Most of the problems were taken from the textbook [1].

Problem 1. Evaluate the indefinite integral as an infinite series.

a) $\int \sqrt{1+x^3} dx$;

b) $\int x^2 \sin(x^2) dx$;

c) $\int \frac{\cos x - 1}{x} dx$;

d) $\int \tan^{-1}(x^2) dx$.

Problem 2. Use series to approximate the definite integral to within the indicated accuracy.

a) $\int_0^{1/2} x^3 \tan^{-1} x dx$ (four decimal places);

b) $\int_0^1 \sin(x^4) dx$ (four decimal places);

c) $\int_0^{0.5} x^2 e^{-x^2} dx$ ($|error| < 0.001$).

Problem 3. Let $f(x) = (1+x^3)^{30}$. Compute $f^{(57)}$ and $f^{(58)}$.

Problem 4. Find the Taylor polynomial $T_3(x)$ for $f(x) = \cos x$ at $a = \pi/2$. Estimate $\cos 80^\circ$ correct to five decimal places.

Problem 5. Approximate $f(x) = x \sin x$ at 0 by its Taylor polynomial $T_4(x)$ when $-1 \leq x \leq 1$. Estimate the potential error of this approximation.

REFERENCES

- [1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.