PROBLEM SET 17: POWER SERIES

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Find the radius of convergence and the interval of convergence of the series.*

a) $\sum_{n=1}^{\infty} n^n x^n$;

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n5^n} x^n$;

c) $\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln n}$;

d) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n}(x+6)^n$;

e) $\sum_{n=1}^{\infty} (2x-1)^n 5^n \sqrt{n}$;

f) $\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n, \quad b > 0$;

g) $\sum_{n=1}^{\infty} n!(3x-1)^n$;

h) $\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$;

i) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{1\cdot 3\cdot 5\cdots(2n-1)}$;

j) $\sum_{n=1}^{\infty} \frac{n! x^n}{1\cdot 3\cdot 5\cdots(2n-1)}$.

Problem 2. *Suppose that* $\sum_{n=0}^{\infty} c_n x^n$ *converges when* $x = -4$ *and diverges when* $x = 6$. *What can we say about the convergence or divergence of the series:*

$$(a)\ \sum_{n=0}^{\infty} c_n; \quad (b)\ \sum_{n=0}^{\infty} c_n 8^n; \quad (c)\ \sum_{n=0}^{\infty} c_n(-3)^n; \quad (d)\ \sum_{n=0}^{\infty} (-1)^n c_n 9^n.$$

Problem 3. *Let* p *and* q *be real numbers such that* $p < q$. *Find a power series whose interval of convergence is (a)* (p,q)*; (b)* $(p,q]$*; (c)* $[p,q)$*, and (d)* $[p,q]$.

Problem 4. *Is it possible to find a power series whose interval of convergence is* $[0,\infty)$*? Explain.*

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.