

## PROBLEM SET 10: SEQUENCES

Note: Most of the problems were taken from the textbook [1].

**Problem 1.** *Decide whether each of the following sequences converges:*

a)  $\{a_n\}$ , where  $a_n = \frac{3\sqrt{n}}{\sqrt{n+2}}$ ;

b)  $\{a_n\}$ , where  $a_n = \ln(n+1) - \ln n$ ;

c)  $\left\{ \frac{\ln n}{\ln 2n} \right\}$ ;

d)  $\{a_n\}$ , where  $a_n = n \sin(1/n)$ ;

e)  $\{a_n\}$ , where  $a_n = \left(1 + \frac{2}{n}\right)^n$ ;

f)  $\{a_n\}$ , where  $a_n = \sqrt[n]{n}$ .

**Problem 2.** *Find the limit of the sequence  $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$ .*

**Problem 3.** *Consider the sequence  $\{a_n\}$  recurrently defined as  $a_1 = 2$  and  $a_{n+1} = \frac{1}{3-a_n}$ .*

- (1) *Assuming that  $0 \leq a_n \leq 2$  and that the sequence  $\{a_n\}$  is decreasing, argue that  $\{a_n\}$  is convergent and find its limit.*
- (2) *Can you show that  $0 \leq a_n \leq 2$  for every natural  $n$ ?*
- (3) *Can you show that  $\{a_n\}$  is decreasing?*

## REFERENCES

- [1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.