MOCK QUIZ 1

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Evaluate the integral*

$$\int_0^1 ze^{2z}\,dz.$$

Solution: We integrate by parts. Taking $u = z$ and $dv = e^{2z}$, we find that

$$du = dz \quad \text{and} \quad v = \int e^{2z}\,dz = \frac{1}{2}e^{2z}.$$

Therefore

$$\int_0^1 ze^{2z}\,dz = \frac{ze^{2z}}{2}\bigg|_0^1 - \frac{1}{2}\int_0^1 e^{2z}\,dz = \frac{e^2}{2} - \frac{e^{2z}}{4}\bigg|_0^1 = \frac{e^2}{4} + \frac{1}{4}.$$

□

Problem 2. *Solve the integral*

$$\int \sin^3 x \cos^4 x\,dx.$$

Solution: First, notice that

$$\begin{aligned}\int \sin^3 x \cos^4 x\,dx &= \int (1-\cos^2 x)\cos^4 x\,\sin x dx \\ &= \int \cos^6 x\,(-\sin x)dx - \int \cos^4 x\,(-\sin x)dx \\ &= \int u^6 du - \int u^4 du \\ &= u^7/7 - u^5/5 + C,\end{aligned}$$

where $u = \cos x$. Going back to the initial variable x, one obtains

$$\int \sin^3 x \cos^4 x\,dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C.$$

□

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.