PROBLEM SET 8: RATES OF CHANGE AND DIFFERENTIALS

Note: Most of the problems were taken from the textbook [1].

Problem 1. *A bacteria culture initially contains* 100 *cells and grows at a rate proportional to its size. After an hour the population has increased to* 420.

(1) Find an expression for the number of bacteria after t *hours.*
(2) Find the number of bacteria after 3 *hours.*
(3) Find the rate of growth after 3 *hours.*
(4) When will the population reach 10000*?*

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Problem 2. *Strontium-*90 *has a half-life of* 28 *days.*

(1) A sample has a mass of 50 *mg initially. Find a formula for the mass remaining after* t *days.*
(2) Find the mass remaining after 40 *days.*
(3) How long does it take the sample to decay to a mass of 2 *mg?*

Problem 3. *A plane flying horizontally at an altitude of* $1mi$ *and a speed of* 500 mi/h *passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2* mi *away from the station.*

Problem 4. *If a snowball melts so that its surface area decreases at a rate of* $1\ \text{cm}^2/\text{min}$*, find the rate at which the diameter decreases when the diameter is* 10 cm.

Problem 5. *A street light is mounted at the top of a* 15-ft*-tall pole. A man* 6 ft *tall walks away from the pole with a speed of* 5 ft/s *along a straight path. How fast is the tip of his shadow moving when he is* 40 ft *from the pole?*

Problem 6. *At noon, ship A is* 150 km *west of ship B. Ship A is sailing east at* 35 km/h *and ship B is sailing north at* 25 km/h*. How fast is the distance between the ships changing at* 4 : 00 pm*?*

Problem 7. *Find the linearization* $L(x)$ *of the functions*

$$f(x) = \sin x \text{ at } \pi/6 \quad \text{and} \quad g(x) = \sqrt{x} \text{ at } 4.$$

Problem 8. *Find the differential of the functions* $f(x) = \ln(\sin x)$ *and* $g(x) = \frac{e^x}{1-e^x}$.

Problem 9. *Estimate* $\sqrt[4]{16.1}$ *and* $\cos 29.5°$.

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.